The notion of group was mentioned in class today. One way to define this for matrices is the following:

**Definition.** A (nonempty) set $S$ of $n \times n$ matrices is said to be a (matrix) group if the following two conditions hold:

i) If $A, B \in S$, then $AB \in S$ (i.e. $S$ is closed under products), and

ii) If $A \in S$, then $A^{-1} \in S$ (i.e. $S$ is closed under inverses. In particular, this implies that all matrices in $S$ are invertible.)

Note: it follows from (i) and (ii) that if $S$ is a matrix group, then the identity matrix must be in $S$.

Show that the following sets of $n \times n$ matrices are groups:

1) $S = \{\text{all invertible matrices}\}$

2) $S = \{\text{all unitary matrices}\}$

3) $S = \{\text{all diagonal matrices with nonzero diagonal entries}\}$

There are many more examples of matrix groups (in fact, every finite group is a matrix group). We will see some more examples later in the course.

Problems in Axler:

7.C: #2, 5, 9

7.D: #2, 5, 6, 10, 15