1. Show that if a rational function is symmetric, then it is a quotient of two symmetric polynomials.

2. Let \( n \in \mathbb{N} \) be odd.
   i) If \( K/\mathbb{Q} \) is finite, show that \( K \) contains a primitive \( n \)th root of 1 iff \( K \) contains all \( (2n) \)th roots of 1.
   ii) Show that \( i = \sqrt{-1} \notin \mathbb{Q}(\zeta_n) \).

3. Show that in a finite field, every element can be written as a sum of exactly two squares.

4. Let \( G \) be a finite abelian group, and \( p \) a prime dividing \(|G|\). Show that for all \( n \gg 0 \), \( \mathbb{Z}/p^n \mathbb{Z} \otimes \mathbb{Z}G \) is isomorphic to the Sylow \( p \)-subgroup of \( G \).