1. Give the Taylor series of 

\[
\sin(x) : \quad \frac{1}{\sqrt{1 - x}} = (1 - x)^{-1/2} : 
\]

2. Give the Taylor expansion up to order \( n \) of (with the explicit expression of the remainder)

\[
\frac{1}{1 - x^2} = 
\]
3. Compute

(a) \[ \lim_{n \to \infty} (n+a)^{1/3}(n+b)^{2/3} - n = \]

(b) \[ \lim_{n \to \infty} \left( \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n} \right) = \]

(c) for \( a > 1 \) and \( \alpha > 1 \)

\[ \lim_{n \to \infty} \frac{a^{n\alpha}}{n!} = \]

(d) Is the following series convergent and why?

\[ \sum_{n>2} \frac{1}{n \ln^{3/2} n} : \]

(e) Is the following series convergent and why?

\[ \frac{1}{\ln(2)} - \frac{1}{\ln(3)} + \frac{1}{\ln(4)} + \cdots + (-1)^n \frac{1}{\ln(n)} + \cdots \]
4. (a) Shows that the integral $\int_0^\infty dy \ e^y / y^y$ converges (Hint: compare with the series $\sum_{n=1}^{\infty} (e/n)^n$)

(b) Using the integral test, is this series convergent or not?

$$\sum_{n=1}^{\infty} \frac{1}{(\ln n)^{\ln n}}$$

5. Show that the following series converges uniformly on $\mathbb{R}$

$$\sum_{n=1}^{\infty} \frac{\sin (3^n x)}{2^n}$$