Some New Topics in International Trade Theory

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One of the oldest theories in economics

- Mercantilists (16-18th centuries)
- Adam Smith (free trade)
- Ricardo: Comparative advantage theory
- Oppositions: Alexander Hamilton, Frederick List (German Historical School)

D. Ricardo’s theory of international trade

- Ricardo succeeded to explain gains from trade even in the case when a country has inferior production techniques than the other in all industries. (Absolute advantage vs. comparative advantage)
- S. Ulam once asked S. if any economic theory is true but not trivial.
- Samuelson’s answer: Ricardo’s theory of comparative advantage
New Interpretation!?

- A new interpretation in 2002-04.
  - Faccarello (2015) Comparative Advantage

- Comparative advantage vs. comparative cost
  - Comparative advantage is not defined in a general case (with input trade).
  - Cost comparison is still valid. Shiozawa (2016b)
Ricardian trade economy

Traditionally, dealt with cases
- $M$-country, $N$-commodity
  - Minimal model was (2, 2) type.
- Production: labor input economy
- Capital goods.
  - Vertical integration
  - Applicable if no input goods are traded.

Explored in 1950’s.
- L. McKenzie, R. Jones
- Crucial defects: No input trade (intermediate goods)
Ricardo-Sraffa trade economy

- $M$-country, $N$-commodity case
- Production: material input
  - Capital: general name of input goods
  - Choice of production techniques
- Input trade (traded intermediate goods)
  - finished goods vs. intermediate goods
  - Introduction of trade in intermediate product necessitates a fundamental alteration of the theory.
  - Distinction between Ricardian t.e. and RS t.e. crucial.
  - Subtropical theory only applies to R.t.e.
Importance of RS trade economy

- The real RS t.e. is structurally different from R t.e.
  - A challenging problem for tropical theory.
  - N.B. If input goods are not traded, RS t.e. is reduced to R. t.e.

- Actual problems are related to RS t.e.
  - Industrial revolution in Lancashire, Cotton.
  - Fragmentation, Global value chain, etc.
Ricardian trade theory as subtropical convex geometry

● Subtropical algebra

- $\mathbb{R}_+$
- $a \oplus b = \text{min}\{a, b\}$, $a \odot b = a \cdot b$
- Isomorphic to the tropical (min, plus)-algebra in $\mathbb{R}$.
- $\log: \mathbb{R}_+ \rightarrow \mathbb{R}; \log(a \cdot b) = \log(a) + \log(b)$.

● Well adapted to the description and analysis of R t.e.

● Details: Shiozawa (2012; 2015a)

● Many topics to be developed.

● Good concrete model of tropical geometry.
Bird’s-eye view of the theory

Tropical polytopes

Subtropical polytopes

Covering Range Rp

World Production Frontier

Price Simplex $\Delta^{n-1}$

Cephoid

Log-lifting over $\Delta^{m-1} \times \Delta^{n-1}$

Caley trick

World pp set

Mixed subdivisions of $n\Delta^{m-1}$

McKenzie-Minabe diagram

Tropical oriented matroid

Triangulations of $\Delta^{m-1} \times \Delta^{n-1}$

Tropical h-plane Arrangement

Sharing Range Rw

Fine type
Ricardian trade economy: mathematical formulation

- **Input coefficient matrix** $A = (a_{ij})$
  - M-row N-column matrix
  - $a_{ij}$ labor input coefficient in country $i$ to produce product $j$

- **Labor power** $q = (q_i)$

- **International value** $v = (w, p)$
  - $w = (w_i)$ wage rate for country $i$
  - $p = (p_j)$ price for product $j$
Some notions: PPS, value, competitive pattern

- **Production possibility set (PPS)**, a polytope in $\mathbb{R}^N_+$.  
  \[ P = \{ y \mid y_j = (\sum_i s_{ij}), \sum_j s_{ij} a_{ij} \leq q_i, s_{ij} \geq 0 \ \forall i \} \]

- **$v = (w, p)$**  
  - $w_i$ wage for labor of country $i$, $i = 1, 2, \ldots, M$.  
  - $p_j$ price of commodity $j$, $j = 1, 2, \ldots, N$.

- **Admissible value** $v = (w, p) > 0$:  
  No $(i,j) w_i a_{ij} < p_j$ (No production with extraordinary profits)

- **Competitive pattern**  
  $t = \{(i, j) \mid w_i a_{ij} = p_j\}$
Main theorem

- At each facet of PP set there exists an admissible international value $\mathbf{v} = (\mathbf{w}, \mathbf{p})$ with $\mathbf{p}$ that is perpendicular to the facet and satisfies equality:
  \[
  <\mathbf{w}, \mathbf{q}> = <\mathbf{p}, \mathbf{y}>
  \]
  where $\mathbf{y}$ is a point in the facet.

- Competitive pattern of a facet is spanning. The converse is true.
A Minimal Model of the Ricardian Trade Theory

(2 country 3 product case)

Domain 1 v1

\{11, 12, 13, 22\}  
(A123 B2)

Domain 2 v2

\{11, 13, 22, 23\}  
(A13 B23)

Domain 3 v3

\{11, 21, 22, 23\}  
(A1 B123)

Ridge 1

Ridge 2
Wage simplex

If we shift to wage simplex:

Segment that connects two points $w_1$ and $w_3$

Originally enigmatic!
A spanning core in a wage simplex: (3,3) Ricardian economy case
Price simplex

This is in reality a variant of tropical hyperplane arrangement.

Each facet has a different competitive type.
An arrangement in $\mathbb{TP}^2$

Figure 1 (Ardila & Develin 2004, p.3)

(3,3,3)  (23,13,3)  2  (2,123,3)

(3,1,3)  1  (123,1,3)  (2,2,3)

(1,1,3)  (2,1,3)  (2,12,13)  3  (2,2,123)

(1,1,1)  (12,1,13)

(2,1,1)  (2,2,1)

(2,2,2)
Why subtropical algebra?

- **Subtropical semiring**
  - \( a \oplus b = \min\{a, b\} \) \( a \odot b = a \cdot b \)

- **Ricardian trade theory**
  - Minimum, multiplication (value relations)
  - Minkowski sum (quantity relations)
  - A natural object for (sub)tropical analysis
  - A concrete object for duality

- **Matrix operation**
  - \( w \boxtimes A = \min_i w_i a_{ij} \) is comparable with \( p_j \)
  - \( v \) is admissible \( \Leftrightarrow w \boxtimes A = p \)
Some new ideas (in economics)

- What happens in the interior of PPS?
  - Economically, this is to investigate unemployment.
  - This requires study admissible value independent of production point.
  - Normal value (main theorem, spanning type)

- Tropical oriented matroid:
  - a set of fine types (⇔competitive types)
Necessary labor set

- A and d are given;
- \( L = \{ q \mid q_i = (\sum_j s_{ij} a_{ij})_i, \sum_i s_{ij} = d_j \} \)
- An admissible value gives upper facet.
- An anti-admissible value gives lower facet.  \( w_{ij} a_{ij} \leq p_j \ \forall \tau = (i,j) \).
- Other values: mixed value
  - \( \exists l,j \ s.t. w_{ij} a_{ij} < p_j \) and \( \exists h,k \ s.t. w_{hk} a_{hk} > p_k \)
Spanning type determines value.

- $A = (a_{ij})$ is given.
  \[ v = (w, p) \Rightarrow T = \{ \tau = (i, j) \mid w_i a_{ij} = p_j \} \]
- $T$: $(M,N)$ bipartite graph $T \in K_{M,N}$
- $T$: spanning tree
  - connected (tree: one connected component)
  - spanning (edges cover all countries and goods)
  - no cycle (no cyclic chain of edges)
  - In (2,3) trade economy, there are 12 different spanning trees. See the next sheet.
Properties of spanning trees and value determination

- $(M,N)$ spanning tree has $M+N-1$ edges.
- Contains leaves (vertex with degree 1)
- Start by any value from a vertex of a leaf $w_i$ if country vertex $i$ and $p_j$ if product vertex $j$.
- Continue fixing the value of a new vertex by eq. $w_i \cdot a_{ij} = p_j$ when $(i,j) \in T$.
- All vertices are covered (spanning) and no contradiction (no cycle)
Matrix $A$ in a general position

- Pallaschke and Rosenmüller (2004), $\mathcal{E} = \{A, q\}$ as a cephoid.
  - Cephoid is a PP set for a Ricardian trade economy.
  - Definition 1.5 ( "nondegenerate" or "in general" position) is rather complicated.

- A new definition:
  - $A$ is in a general position $\iff$
    \[ T = \{(i,j) \mid w_i a_{ij} = p_j\} \text{ is acyclic } \forall w, p. \]
  - We may restrict the range of definition to normal values.
Bipartite graph corresponding to directed 2,3 Ricardian trade economy $K_{2,3}$

An example of closed cycle: A2B3A
An acyclicity theorem

Theorem:
If T1 and T2 are two different competitive types of a matrix A in general position, then directed bipartite graph $T_1 \cup T_2$ has no directed cycle.

Proof: Let $v_1$ and $v_2$ be values determined respectively by T1 and T2. If cycle exists, $v_1 = v_2$ or matrix A is not in general position. QED.
Types that can be consistent
Problems:

- Number of spanning trees for bipartite graph $M^{N-1} \cdot N^{M-1}$ (Scoin’s formula).
- Set of normal types
  - (A1 B123), (A13 B23), (A123 B2)
  - Number of normal types: $\binom{M+N-1}{M}$ ?
- Can we characterize the set of normal types that may corresponds to a matrix?
- How any spanning types in a given class?
Really challenging problems:

- Can we extend the theory to RS trade economy? (RS is much more important than R)

Value relation:
\[ \min \{ w_{i0} + a_{i1} p_1 + \cdots + a_{iN} p_N \} = p_N \]

Tropical parallelism:
\[ \bigoplus \{ w_{i0} \odot p_1^{a_1} \odot \cdots \odot p_N^{a_N} \} = p_N \]

Here \( a_{ij} \) can be assumed integral, but very large.
References:

- Shiozawa (2012) Subtropical convex geometry as the Ricardian theory of international trade, in RG.
- Shiozawa (2016) New Interpretation of Ricardo’s four Magic Numbers and the New theory of International Values, in RG.
Thank you.

- Questions and comments welcome.
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