Some New Topics in International Trade Theory

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Backgound for Ricardian trade theory

- One of the oldest theories in economics
  - Mercantilists (16-18th centuries)
  - Adam Smith (free trade)
  - Ricardo: Comparative advantage theory
  - Oppositions: Alexander Hamilton, Frederick List (German Historical School)
- D. Ricardo’s theory of international trade
  - *Principles of Political Economy and Taxation,* 1817. Chap.7 On Foreign Trade.
  - Ricardo succeeded to explain gains from trade even in the case when a country has inferior production techniques than the other in all industries. (Absolute advantage vs. comparative advantage)
  - S. Ulam once asked S. if any economic theory is true but not trivial.
  - Samuelson’s answer: Ricardo’s theory of comparative advantage

New Interpretation!?

- A new interpretation in 2002-04.
  - Faccarello (2015) Comparative Advantage
  - Shiozawa (2016) The New Interpretation of Ricardo's Four Magic Numbers and the New Theory of International Values, in RG.
- Comparative advantage vs. comparative cost
  - Comparative advantage is not defined in a general case (with input trade).
  - Cost comparison is still valid. Shiozawa (2016b)

Ricardian trade economy

- Traditionally, dealt with cases
  - *M*-country, *N*-commodity
    - Minimal model was (2, 2) type.
  - Production: labor input economy
  - Capital goods.
    - Vertical integration
    - Applicable if no input goods are traded.
- Explored in 1950's.
  - L. McKenzie, R. Jones
  - Generalization: Dornbusch, Fischer, and Samuelson (1977) Case of a Continuum of goods.
  - Crucial defects: No input trade (intermediate goods)

Ricardo-Sraffa trade economy

- *M*-country, *N*-commodity case
- Production: material input
  - Capital: general name of input goods
  - Choice of production techniques
- Input trade (traded intermediate goods)
  - finished goods vs. intermediate goods
  - Introduction of trade in intermediate product necessitates a fundamental alteration of the theory.
  - Distinction between Ricardian t.e. and RS t.e. crucial.
  - Subtropical theory only applies to R.t.e..

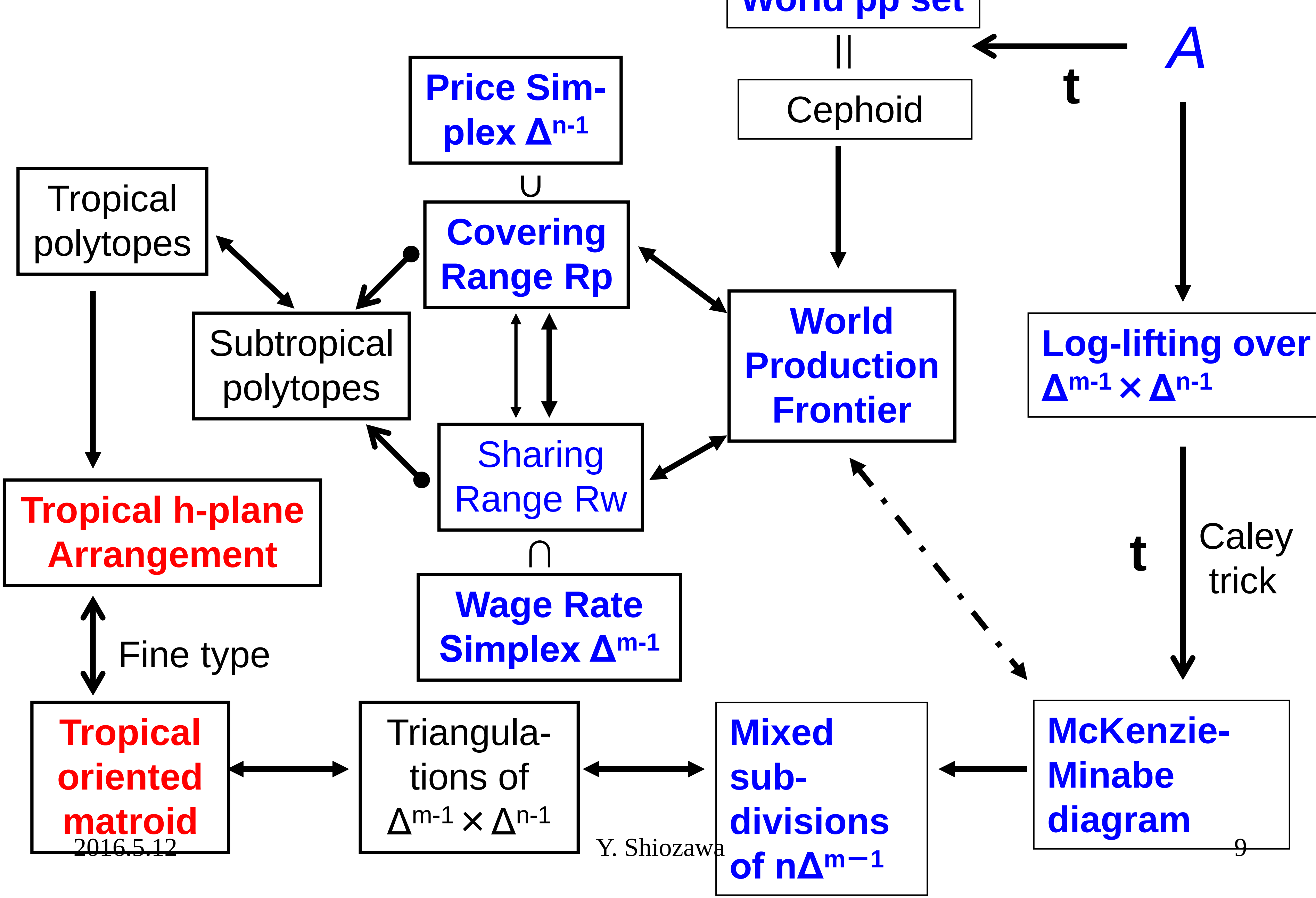
Importance of RS trade economy

- The real RS t.e. is structurally different from R t.e.
  - A challenging problem for tropical theory.
  - N.B. If input goods are not traded, RS t.e. is reduced to R. t.e.
- Actual problems are related to RS t.e.
  - Industrial revolution in Lancashire, Cotton.
  - Fragmentation, Global value chain, etc.

Ricardian trade theory as subtropical convex geometry

- Subtropical algebra
  - $\mathbb{R}_+$
  - $a \oplus b = \min\{a, b\}$ $a \odot b = a \cdot b$
  - isomorphic to the tropical (min, plus)-algebra in $\mathbb{R}$.
  - log: $\mathbb{R}_+ \to \mathbb{R}$; $\log (a \cdot b) = \log (a) + \log (b)$.
- Well adapted to the description and analysis of R t.e.
- Details: Shiozawa (2012; 2015a)
- Many topics to be developed.
- Good concrete model of tropical geometry.

Bird's-eye view of the theory



Ricardian trade economy: mathematical formulation

- Input coefficient matrix $A = (a_{ij})$
  - M-row N-column matrix
  - a_{ij} labor input coefficient in country i to produce product j
- labor power $\mathbf{q} = (q_i)$
- International value $\mathbf{v} = (\mathbf{w}, \mathbf{p})$
  - $\mathbf{w} = (w_i)$ wage rate for country i
  - $\mathbf{p} = (p_j)$ price for product j

Some notions: PPS, value, competitive pattern

- Production possibility set (PPS), a polytope in $\mathbb{R}_+^N$.

  $P = \{\mathbf{y} \mid y_j = (\sum_i s_{ij}), \sum_j s_{ij}\, a_{ij} \leqq q_i,\ s_{ij} \geqq 0\ \forall i\}$

- $\mathbf{v} = (\mathbf{w}, \mathbf{p}) = (w_1, \ldots, w_M, p_1, \ldots, p_N)$
  - w_i wage for labor of country i, $I = 1, 2, \ldots, M$.
  - p_j price of commodity j, $j = 1, 2, \ldots, N$.
- Admissible value $\mathbf{v} = (\mathbf{w}, \mathbf{p}) > \mathbf{0}$:

  No (i,j) $w_i\, a_{ij} < p_j$ (No production with extraordinary profits)

- Competitive pattern $t = \{(i, j) \mid w_i\, a_{ij} = p_j\}$

Main theorem

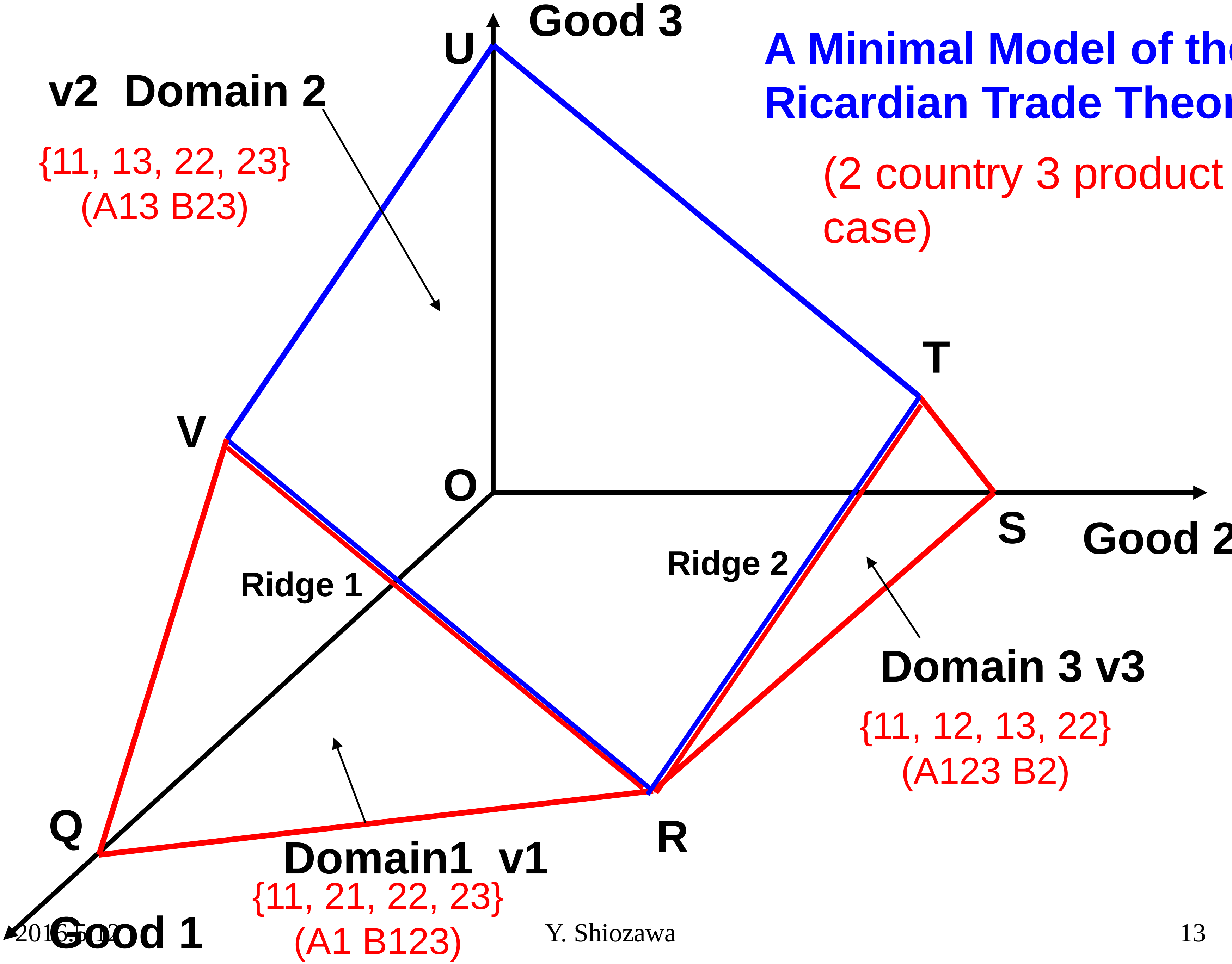
- At each facet of PP set there exists an admissible international value $\mathbf{v} = (\mathbf{w}, \mathbf{p})$ with $\mathbf{p}$ that is perpendicular to the facet and satisfies equality:

$$\langle \mathbf{w}, \mathbf{q} \rangle = \langle \mathbf{p}, \mathbf{y} \rangle$$

  where $\mathbf{y}$ is a point in the facet.
- Competitive pattern of a facet is spanning. The converse is true.

A Minimal Model of the Ricardian Trade Theory

(2 country 3 product case)

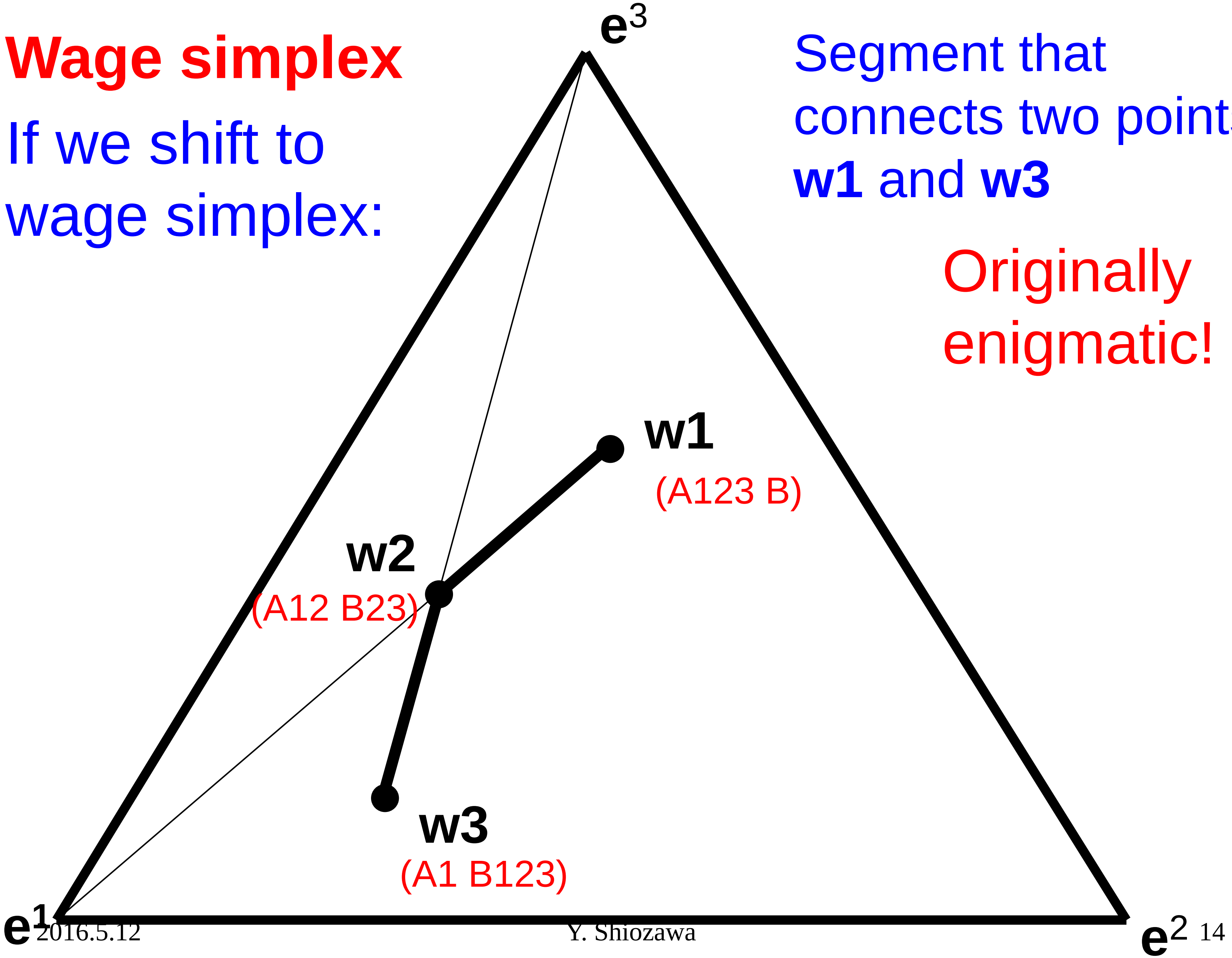


Wage simplex

If we shift to wage simplex:

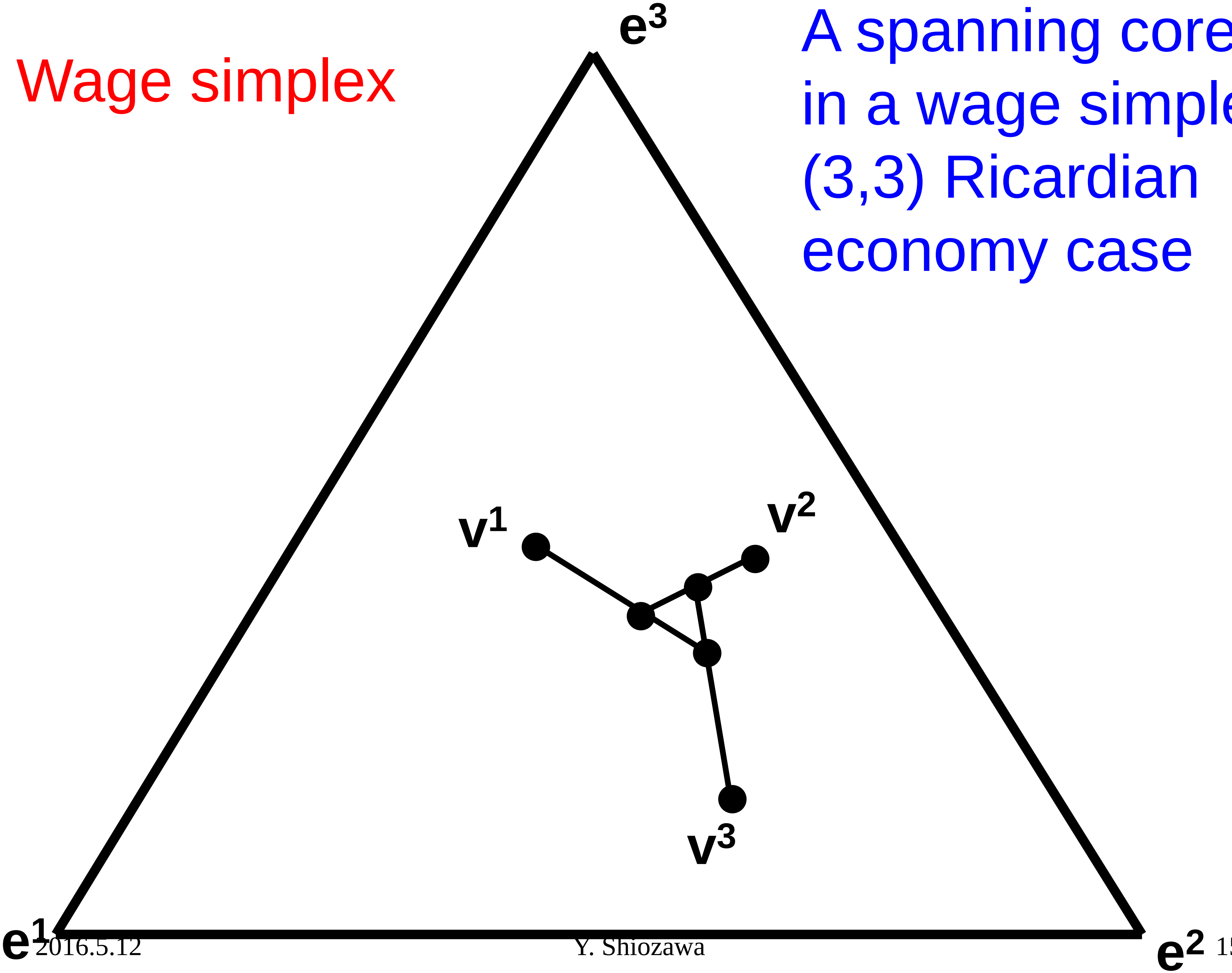
Segment that connects two points **w1** and **w3**

Originally enigmatic!



Wage simplex

A spanning core
in a wage simplex:
(3,3) Ricardian
economy case

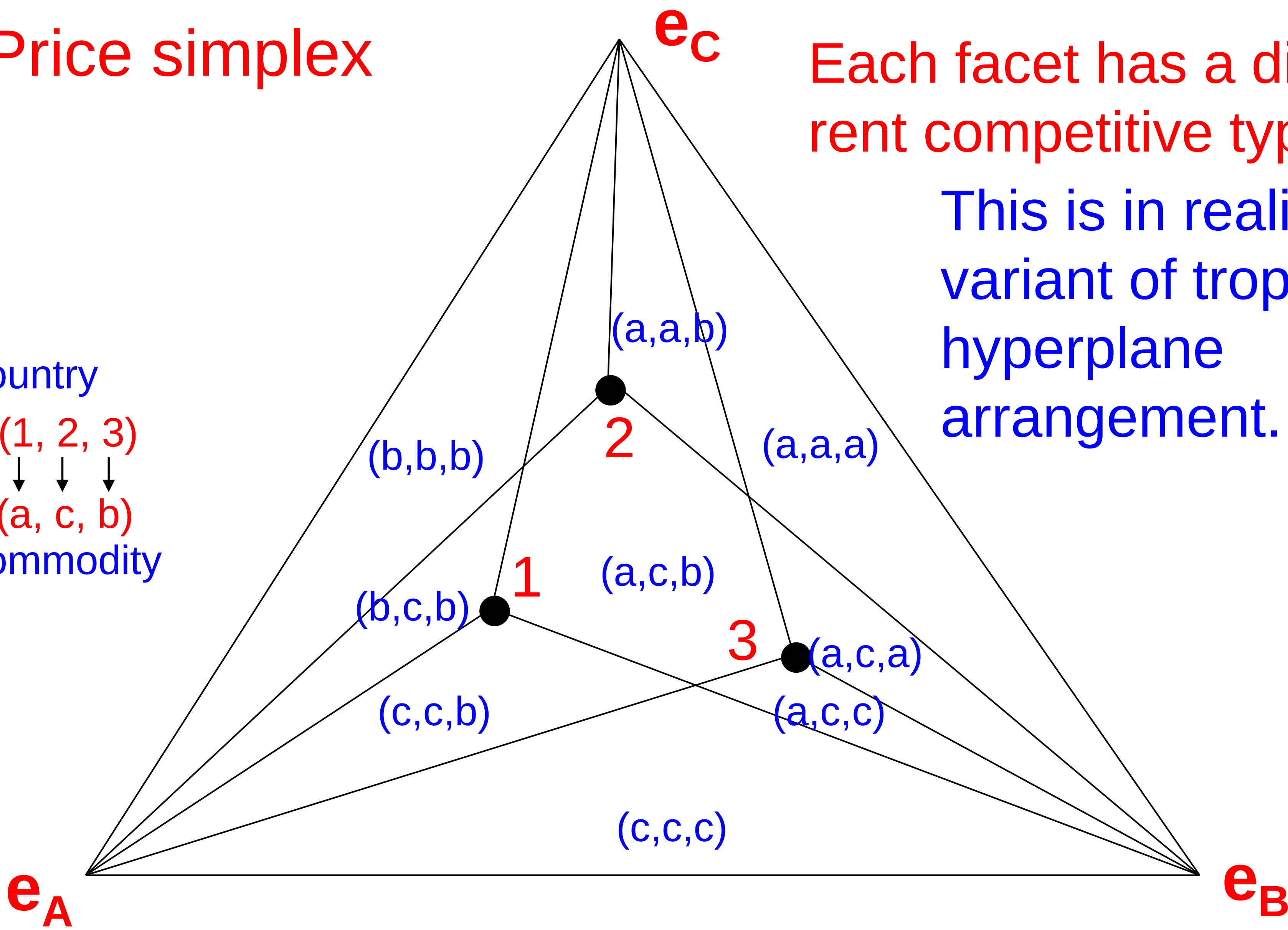


Price simplex

Each facet has a diffe-rent competitive type.

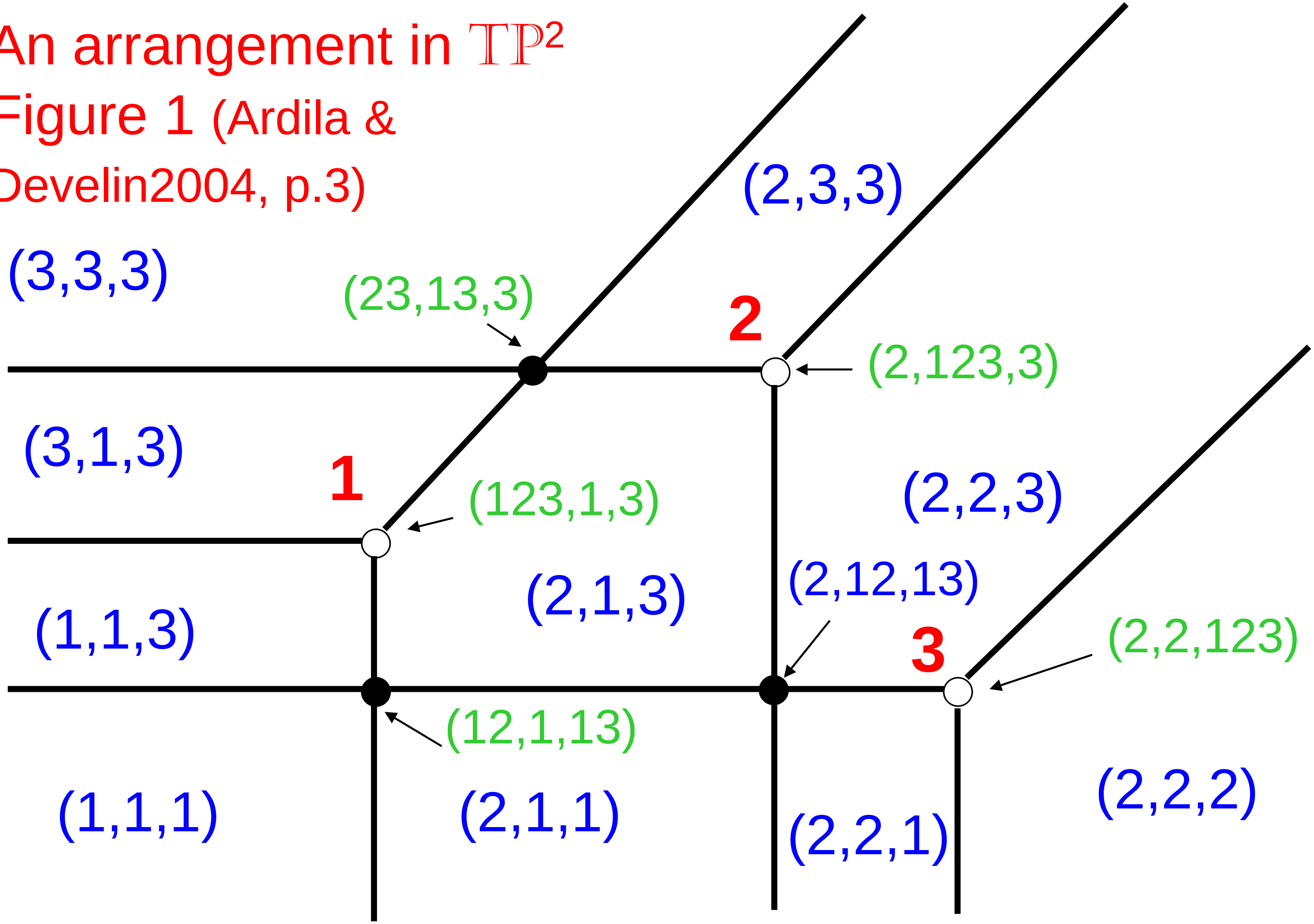
This is in reality a variant of tropical hyperplane arrangement.

Country
(1, 2, 3)
↓ ↓ ↓
(a, c, b)
Commodity



An arrangement in $\mathbb{TP}^2$

Figure 1 (Ardila & Develin2004, p.3)



Why subtropical algebra?

- Subtropical semiring
  - $a \oplus b = \min\{a, b\}$ $a \odot b = a \cdot b$
- Ricardian trade theory
  - Minimum, multiplication (value relations)
  - Minkowski sum (quantity relations)
  - A natural object for (sub)tropical analysis
  - A concrete object for duality
- Matrix operation
  - $w \otimes A = \min_i w_i a_{ij}$ is comparable with p_j
  - **v** is admissible $\Leftrightarrow$ $w \otimes A = p$

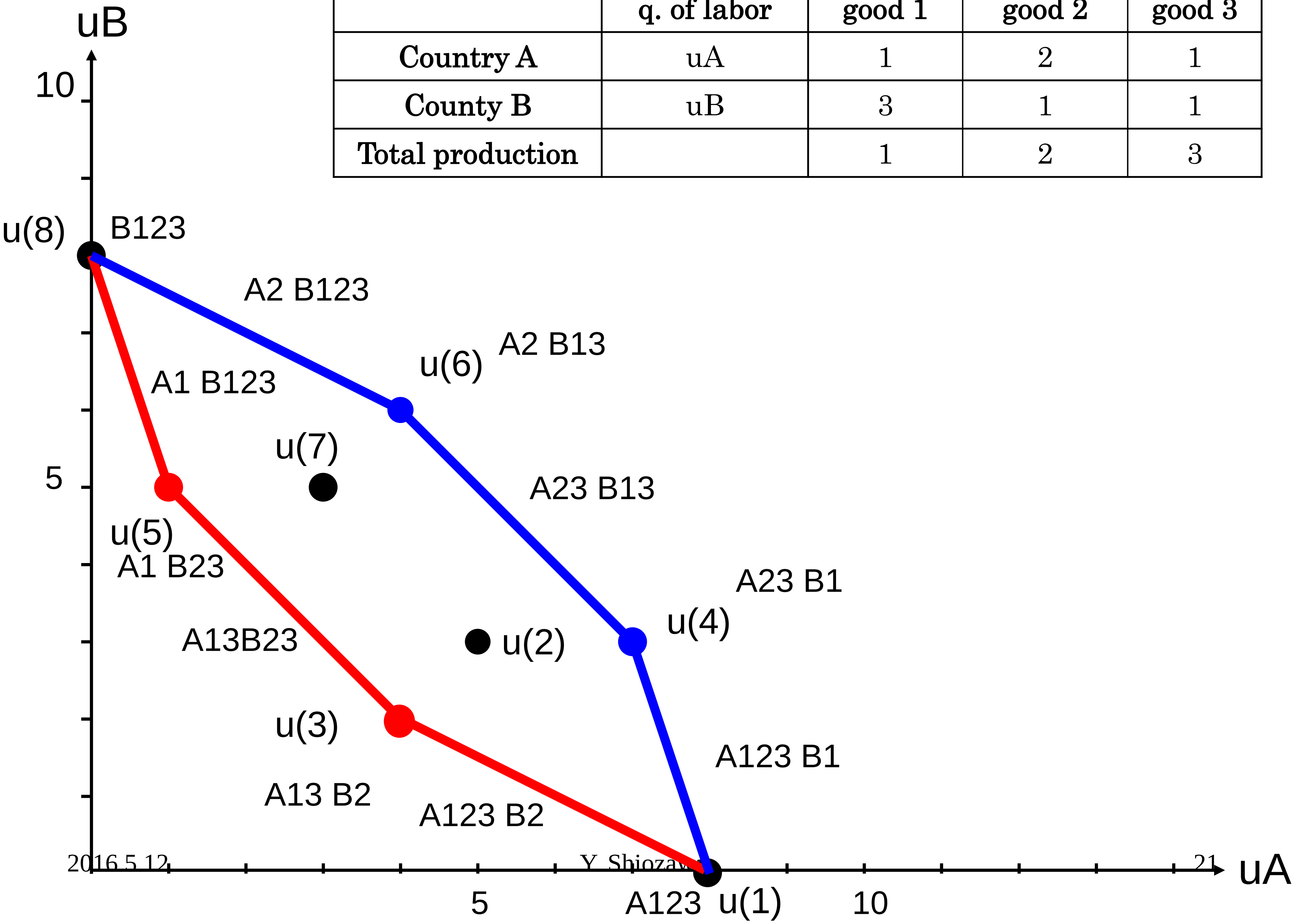
Some new ideas (in economics)

- What happens in the interior of PPS?
  - Economically, this is to investigate unemployment.
  - This requires study admissible value independent of production point.
  - **Normal value** (main theorem, spanning type)
- Tropical oriented matroid:
  - a set of fine types (⇔competitive types)

Necessary labor set

- A and d are given;
- L = { q | $q_i = (\sum_j s_{ij} a_{ij})_i$, $\sum_i s_{ij} = d_j$}
- An admissible value gives upper facet.
- An anti-admissible value gives lower facet. $w_{ij} a_{ij} \leqq p_j \;\; \forall \tau=(i,j)$.
- Other values: mixed value
  - $\exists I,j \;\; wij \; aij < pj$ and $\exists h,k \; w_{hk} \, a_{hk} > p_k$

	q. of labor	good 1	good 2	good 3
Country A	uA	1	2	1
County B	uB	3	1	1
Total production		1	2	3



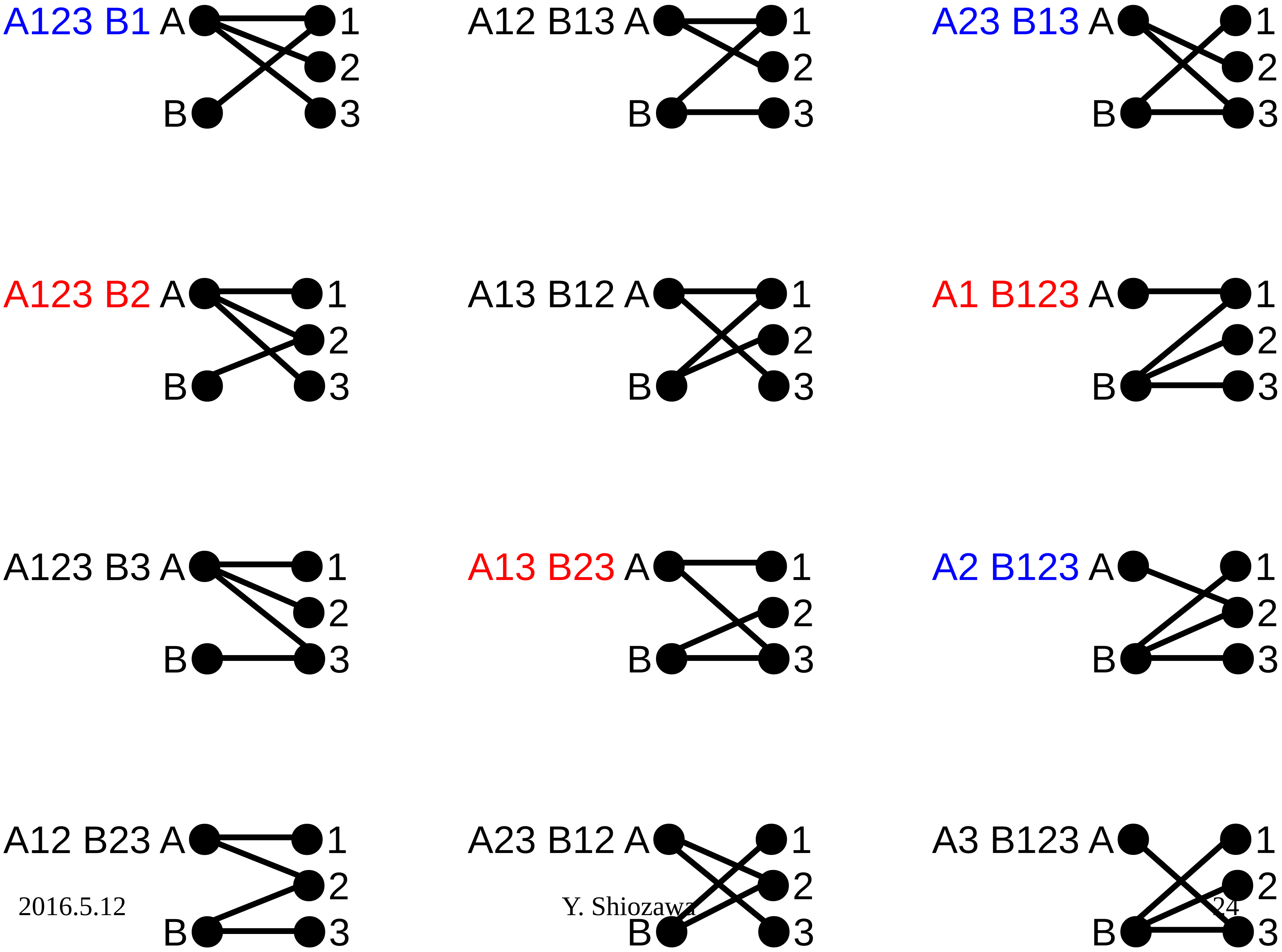
Spanning type determines value.

- $A = (a_{ij})$ is given.

  $\mathbf{v} = (\mathbf{w}, \mathbf{p}) \Rightarrow T = \{\tau=(i,j) | \; w_i \, a_{ij} = p_j \}$
- T: (M,N) bipartite graph $T \in K_{M,N}$
- T: spanning tree
  - connected (tree: one connected component)
  - spanning (edges cover all countries and goods)
  - no cycle (no cyclic chain of edges)
  - In (2,3) trade economy, there are 12 different spanning trees. See the next sheet.

Properties of spanning trees and value determination

- (M,N) spanning tree has M+N-1 edges.
- Contains leaves (vertex with degree 1)
- Start by any value from a vertex of a leaf w_i if country vertex i and p_j if product vertex $_j$.
- Continue fixing the value of a new vertex by eq. $w_i a_{ij} = p_j$ when $(i,j) \in T$.
- All vertices are covered (spanning) and no contradiction (no cycle)


A123 B1
A12 B13
A23 B13
A123 B2
A13 B12
A1 B123
A123 B3
A13 B23
A2 B123
A12 B23
A23 B12
A3 B123
A
B
1
2
3

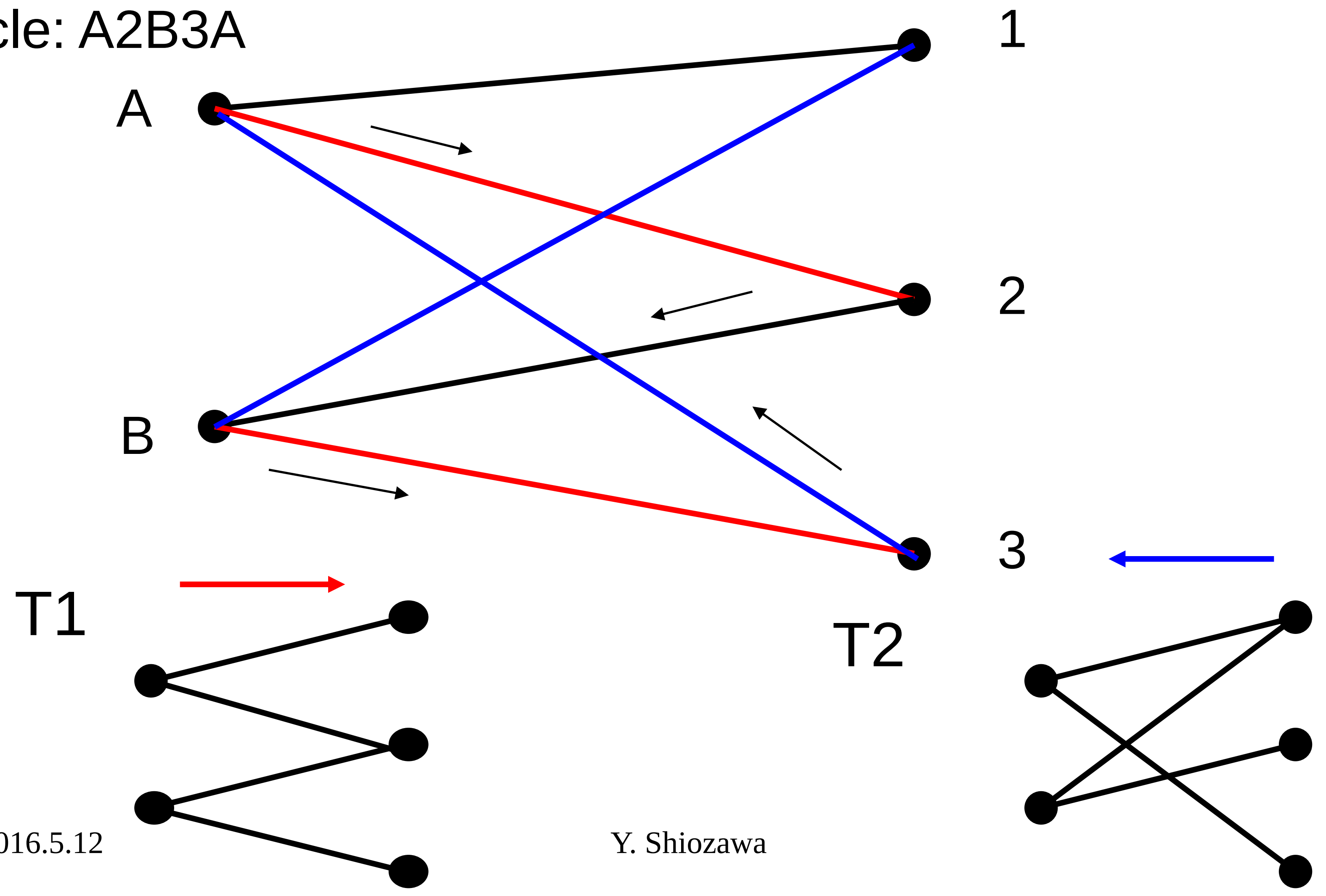
Matrix A in a general position

- Pallaschke and Rosenmüller (2004), $\mathcal{E} = \{A, \mathbf{q}\}$ as a cephoid.
  - Cephoid is a PP set for a Ricardian trade economy.
  - Definition 1.5 (“nondegenerate” or “in general” position) is rather complicated.
- A new definition:
  - A is in a general position ⇔

    $T = \{(i,j) \mid w_i\, a_{ij} = p_j\}$ is acyclic ∀w, p.
  - We may restrict the range of definition to normal values.

Bipartite graph corresponding to directed 2,3 Ricardian trade economy $K_{2,3}$

An example of closed cycle: A2B3A



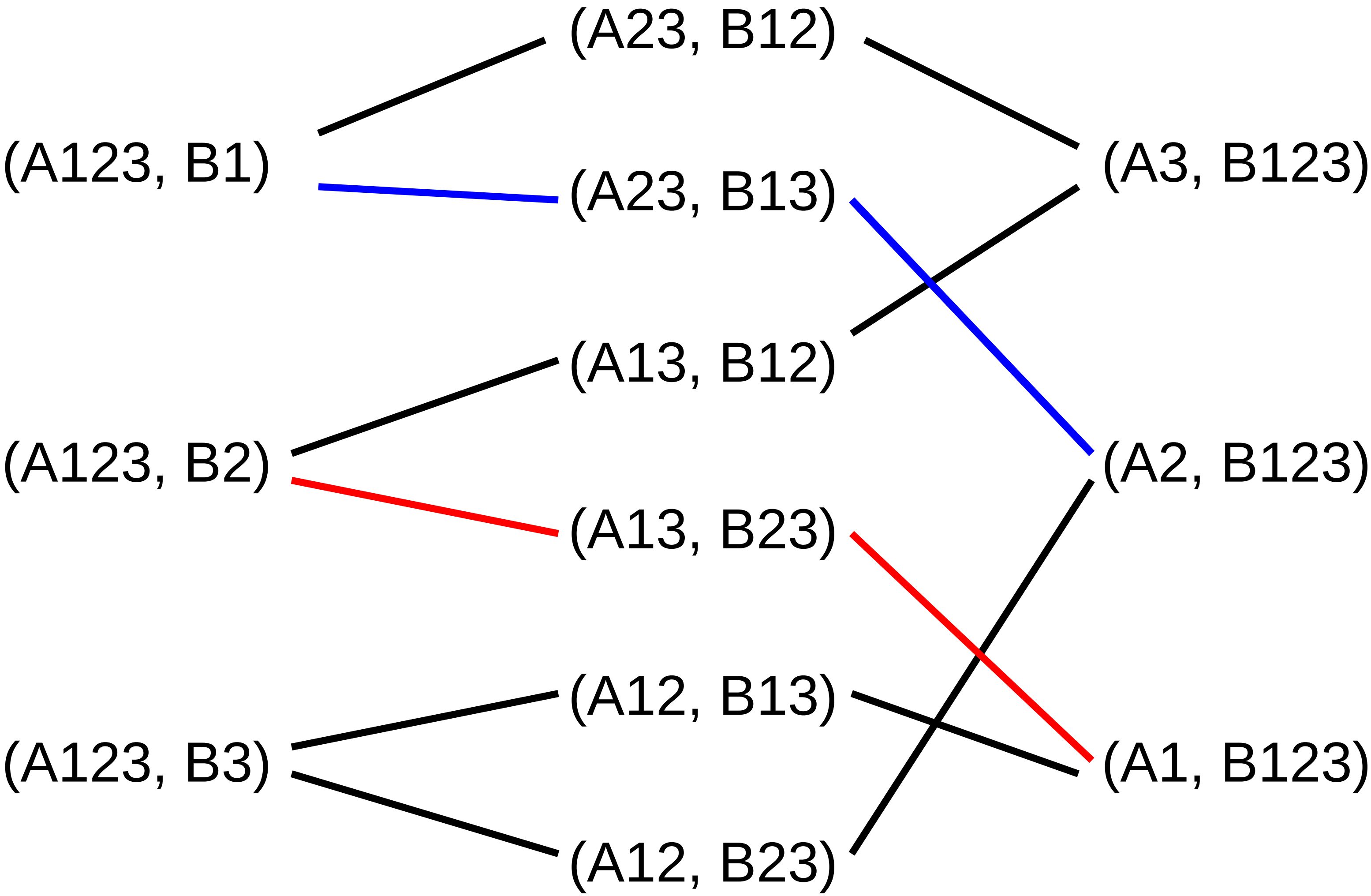
An acyclicity theorem

Theorem:

If T1 and T2 are two different competitive types of a matrix *A* in general position, then directed bipartite graph T1∪T2 has no directed cycle.

Proof: Let v1 and v2 be values determined repectively by T1 and T2. If cycle exists, v1 = v2 or matrix *A* is not in general position. QED.

Types that can be consistent



Problems:

- Number of spanning trees for bipartite graph $M^{N-1} \cdot N^{M-1}$ (Scoin's formula).
- Set of normal types
  - (A1 B123), (A13 B23), (A123 B2)
  - Number of normal types: ${}_{M+N-1}C_{M}$?
- Can we characterize the set of normal types that may corresponds to a matrix?
- How any spanning types in a given class?

Really challenging problems:

- Can we extend the theory to RS trade economy? (RS is much more important than R)

value relation:

$$\min\{w_{i0} + a_{i1}\, p_1 + \cdots + a_{iN}\, p_N\} = p_N$$

tropical parallelism:

$$\bigoplus\{w_{i0} \odot p_1^{a_{i1}} \odot \cdots \odot p_N^{a_{iN}}\} = p_N$$

Here

a_{ij} can be assumed integral, but very large.

References:

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- Shiozawa (2015b) International values and associated spanning trees (in Japanese).
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Thank you.

- Questions and comments welcome.
- Write to y@shiozawa.net