Working Group for Problems in Transport and Related Topics in Graphs

[Tuesday, May 9, 2017]

10:05 – 10:55am : Stan Osher (UCLA)

Title: **L1 Monge-Kantorovich Problem with Applications**
joint with W. Li, W. Gangbo, Y. Li, E. Ryu, B. Wang P. Yin and W. Yin

Abstract: We develop fast methods for solving L1 Monge-Kantorovich type problems using techniques borrowed from fast algorithms for L1 regularized problems arising in compressive sensing. Our method is very simple, easy to parallelize and can be easily combined with other regularizations. We use it in applications including partial optimal transport, image segmentation, image alignment and others. It is flexible enough to easily deal with histograms and other features of the data.

11:05 – 11:55am : Max Fathi (CNRS, Lyon, France)

Title: **Functional inequalities for Markov chains with nonnegative curvature**

Abstract: In this talk, I will present results obtained with Matthias Erbar on functional inequalities for Markov chains on discrete spaces with nonnegative entropic Ricci curvature and bounded diameter, which are discrete analogs of classical results of Riemannian geometry by Li and Yau, Buser and Wang.

1:30 – 2:20pm : Paul-Marie Samson (U. of Paris-Est, Marne La Vallee)

Title : **A generalized Kantorovich-dual theorem and its applications to concentration.**

Abstract: We present a generalization of the classical Kantorovich-dual Theorem that involves new type of costs. This general notion of transport cost encompasses many costs used in the literature, including the classical one introduced by Talagrand and Marton in the 90' to get concentration properties for probability measures. As a by-product, this Kantorovich-dual Theorem applies in different directions. It gives a short proof of a result by Strassen on the existence of a martingale with given marginals. It also provides a full description of a large class of transport-entropy inequalities in terms of exponential integrability of some new infimum convolution operators. The new transport costs that we consider are well suited to describe concentration properties of some discrete measures. As examples, we will mention the binomial law, the Poisson measure on Z, and some probability measures on groups of permutations (Ewens distributions).

Based on :

- *Kantorovich duality for general transport costs and applications,* 2015. Joint work with N. Gozlan, C. Roberto et P. Tetali. http://arxiv.org/abs/1412.7480

- *Transport-entropy inequalities on locally acting groups of permutations,* 2016. https://hal.archives-ouvertes.fr/hal-01370491

3:05 – 3:55pm : Wuchen Li (UCLA)

Title: **Schrödinger equation on graphs via optimal transport**

Abstract: In 1966, Nelson derived Schrödinger equation by diffusion process. Nowadays this approach connects with the theory of optimal transport. We consider similar matters on finite graphs. We propose a discrete Schrödinger equation from Nelson's idea and optimal transport. The proposed equation enjoys several dynamical features. Many numerical examples are also presented.

4:05 – 4:55pm : Xiaojing Ye (Georgia State University)

Title: **Influence Prediction for Continuous-Time Information Propagation on Networks Using Graph-Based Fokker-Planck Equation**

Abstract: We consider the problem of predicting influence, defined as the expected number of infected nodes, resulted from information propagating from any given set of source nodes on a network. We develop a novel and transformative framework that adaptively aggregates the activation states of the network according to the number of active nodes, leading to the construction of a system of differential equations that governs the time evolution of the state probabilities. This system is analogous to the Fokker-Planck equation in continuous space, and the solution readily yields the desired influence. This approach gives rise to a class of novel and scalable algorithms that work effectively for large-scale and dense networks. Numerical results on a variety of synthetic and real-world networks will be presented.

[Wednesday, May 10, 2017]

09:30 – 10:20am : Adam Oberman (McGill University)

Title: **Numerical methods for computing Optimal Transportation maps. joint work with Brittany Froese, JD Benamou, and Yuanlong Ruan**

Abstract: The Optimal Transportation problem has been the subject of a great deal of attention by theoreticians in last couple of decades. Until recently, computation of these distances (and the associated maps) has been intractable, except for very small problems.

Current applications of Optimal Transportation include: Freeform Illumination Optics for shaping light or laser beams, Shape Interpolation (in computer graphics), Machine learning (comparing histograms), discretization of nonlinear PDEs (using the gradient flow in the Wasserstein metric), parameter estimation in geophysics, matching problems in mathematical economics, and Density Functional Theory in physical chemistry.

In the special, but important case of quadratic costs, the map can be obtained from the solution of the elliptic Monge-Ampere partial differential equation with nonstandard boundary conditions. For more general costs, the Kantorovich plan can be approximated by a finite dimensional linear program.

The linear programming approach has the advantage that it easily extends to more general problems: partial transportation, barycentre problems, etc. However the solutions computed are very coarse: convergence is weak (in the sense of measures) so oscillations can appear in the solutions. For LP approaches, it remains an open problem to compute accurate maps.

In the PDE approach, the measures are approximated by densities, and map is obtained as the gradient of a convex function. The boundary conditions are defined implicitly by a Hamilton-Jacobi (first order nonlinear partial differential) equation. We use a wide stencil finite difference scheme, which is augmented by a second order accurate finite difference scheme, resulting in a second order method. We prove convergence to the unique viscosity solution.

10:30 – 11:20am : Nicolas Garcia Trillos (Brown University)

Title: **Gromov-Hausdorff limit of Wasserstein spaces on point clouds.**

Abstract: Inferring geometric properties of a ground-truth measure based on the observation of finitely many samples is an important task with applications to machine learning and statistics. Given that many analytical and geometrical notions at the continuum level can be analyzed by interpreting them in Wasserstein space, it is then natural to do the same at the sample level by considering the right notion of discrete Wasserstein space and ask: when and how are these notions stable as the sample size grows to infinity? The main result that I will present in the talk can be used to establish a variety of consistency results for evolutions of gradient flow type that are relevant to tasks like manifold learning and clustering.

11:30 – 12:20 : Michael Loss (Georgia Tech)

Title: **Decay of entropy and the Kac master equation**

Abstract: The Kac master equation models randomly colliding particles. One of the key measures for approach to equilibrium is the decay of entropy. For general initial conditions not much is known about this decrease, except, that the entropy production can be very small, essentially inversely proportional to the size of the system. It turns out, however, that for initial conditions where M particles are out of equilibrium and N particles are in thermal equilibrium one can give quantitative estimates on the rate of decay. The computations require a number of tools from mathematical physics, such as Nelson's hypercontractive estimate and the geometric version of the Brascamp Lieb inequalities. This is joint work with Federico Bonetto, Alissa Geisinger and Tobias Ried.

1:30 – 2:20 : Paul Horn (University of Denver)

Title: **From Heat Flow to Graph Geometry**

Abstract: A classical way of understanding the geometry of a Riemannian manifold via it's curvature is by using a curvature lower bound to understand heat flow and using that to derive geometric consequences. In this talk, we'll discuss some methods developed by the speaker and his collaborators to perform a similar program on graphs. In particular, we'll discuss how one can use the exponential curvature dimension inequality to prove Li-Yau and Hamilton type gradient estimates for the heat equation, and other further developments that enabled us to show that non-negatively curved graphs (per the CDE' curvature lower bound) satisfy volume doubling and a Poincare inequality (along with associated consequences due to Delmotte.)

2:30 – 3:20 : J. D. Walsh (Georgia Tech)

Title: **The boundary method for optimal mass transportation and Wasserstein distance computation.**

Abstract: Numerical optimal transport is an important area of research, but many problems are too large and complex for easy computation. Because continuous transport problems are generally solved by conversion to semi-discrete forms, we developed the boundary method for semi-discrete transport. The boundary method works with unaltered ground cost functions. It rapidly identifies region boundaries: locations in the continuous space where transport destinations change. Because the method concentrates computation over those boundaries, rather than the entire continuous space, it reduces the effective dimension of the discretization. The boundary method is an effective mesh generation method, able to solve many transport problems that are intractable using other approaches. Even where other numerical methods exist, our tests indicate that the boundary method's performance will compare favorably.