August 10-11

GT-MAP Workshop on

Dynamical Systems

Book of titles and abstracts

With gratitude for nearly 30 years service to our community, this workshop is dedicated to our colleague Shui-Nee Chow, on the occasion of his retirement from Georgia Tech.
# Schedule of Workshop

## Thursday August 10

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
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<tbody>
<tr>
<td>8:30-9:00</td>
<td>Coffee</td>
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<tr>
<td>9:00-9:10</td>
<td>Welcome by Paul Goldbart</td>
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<tr>
<td>9:10-9:50</td>
<td>Erik Van Vleck</td>
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<tr>
<td>9:50-10:30</td>
<td>Wenzhang Huang</td>
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<tr>
<td>10:30-11:00</td>
<td>Coffee</td>
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<tr>
<td>11:00-11:25</td>
<td>Wuchen Li</td>
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<tr>
<td>11:25-12:05</td>
<td>Wenxian Shen</td>
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<tr>
<td>12:05-1:30</td>
<td>Lunch</td>
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<tr>
<td>1:30-2:10</td>
<td>John Mallet-Paret</td>
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<tr>
<td>2:10-2:35</td>
<td>Sung Ha Kang</td>
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<tr>
<td>2:35-3:00</td>
<td>Chongchun Zeng</td>
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<tr>
<td>3:00-3:30</td>
<td>Coffee</td>
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<tr>
<td>3:30-4:10</td>
<td>Eric Carlen</td>
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<tr>
<td>4:10-4:35</td>
<td>Sao Carvalho</td>
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<td>4:35-5:00</td>
<td>Rafael de la llave</td>
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## Friday August 11

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<tr>
<td>8:30-9:00</td>
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<tr>
<td>9:00-9:40</td>
<td>Konstantin Mischaikow</td>
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<td>9:40-10:20</td>
<td>Weishi Liu</td>
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<tr>
<td>10:20-10:45</td>
<td>Rachel Kuske</td>
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<tr>
<td>10:45-11:15</td>
<td>Coffee</td>
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<tr>
<td>11:15-11:55</td>
<td>Todd Young</td>
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<tr>
<td>11:55-12:20</td>
<td>Cinzia Elia</td>
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<tr>
<td>12:20-2:00</td>
<td>Lunch</td>
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<tr>
<td>2:00-2:40</td>
<td>Bo Deng</td>
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<td>2:40-3:05</td>
<td>Howie Weiss</td>
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<td>Coffee</td>
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<tr>
<td>3:30-4:10</td>
<td>Kening Lu</td>
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<tr>
<td>4:10-4:35</td>
<td>Evans Harrell</td>
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<tr>
<td>4:35-5:15</td>
<td>Jim Yorke</td>
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Eric Carlen, Rutgers University

Title:
Operator inequalities, Non-commutative Probability and Quantum Statistical Mechanics

Abstract:
This talk provides an elementary introduction to some problems in the intersection of the three active research areas listed in the title. The focus will be on explaining some ideas involving the analysis of "large deviations" that have been very successfully applied to problems in classical statistical mechanics. For quantum statistical mechanics, the non-commutativity of observables introduces many challenges, and a full quantum analog of the mathematical framework that has proven so useful in the classical case is missing. We shall give a simple survey of several possible ways forward and related open problems that are the focus of current research. The talk requires no particular background: The basic physical problem is easily explained to mathematicians, and then the tools we use are linear algebra and convex analysis.
Title:
Quantum Master Equations in Kinetic Theory

Abstract:
We present recent results on models for quantum systems of $N$ particles undergoing random binary collisions, focusing on the rate of convergence to equilibrium and the propagation of chaos. These questions arise from the work of Mark Kac and his investigation into the probabilistic structure underlying the Boltzmann equation. Recently, the quantum mechanical variation on Kac’s question has begun to be investigated. In this case, the Kac Master equation becomes an evolution equation of Lindblad type, while the corresponding Boltzmann equation is a novel sort of non-linear evolution equation for a density matrix. The treatment departs from the classical treatment because in quantum mechanics, conditional probability is not always well defined. Nonetheless, a substantial quantum analog of the Kac program can be carried out, and it leads to an interesting and novel class of quantum kinetic equations.

This is joint work with Eric Carlen and Michael Loss.
Title: Conductance-resistance symmetrical model for neurons

Abstract:

In this talk we will consider the question that if we construct a neuron model by considering the resistances of ion channels will we find the same conductance based model? We will present two classes of such CR symmetrical models. This implies that neuron models can be constructed independent of modeler's biases.
Cinzia Elia, University of Bari (Italy)

Title:
On periodic orbits of discontinuous dynamical systems

Abstract:
We consider an n-dimensional discontinuous dynamical system with an hyperplane of discontinuity $\Sigma$. We assume that the system has a periodic orbit $\gamma$ and establish the stability of $\gamma$ through two complementary tools: the eigenvalues of the monodromy matrix and the attractivity of $\Sigma$.

We show the following:
i) If $\gamma$ is asymptotically stable then it persists under regularization;
ii) the Floquet multipliers of the regularized periodic orbit converge to the Floquet multipliers of $\gamma$ as the regularization parameter goes to 0.

Joint work with Luca Dieci and Dingheng Pi.
Wenzhang Huang,  
University of Alabama-Huntsville

Title:  
Global dynamics of a ratio-dependent Holling-Tanner predator-prey system

Abstract:  
A ratio-dependent Holling-Tanner predator-prey model is studied. By investigating the local stability properties of equilibrium points and periodic solutions of a transformed system, and by applying the Poincare-Bendixson theorem, we are able to give a complete classification of the global dynamics of the ratio-dependent Holling-Tanner predator-prey model.
Wuchen Li,
University of California at Los Angeles

Title:
Dynamical system on finite graphs

Abstract:
Finite dimensional ODEs have strong connections with the
dynamics in the space of densities, in which optimal transport
provides the important Remannian metric structure. We consider
similar matters on finite graphs, focus on gradient and Hamiltonian
flows. Various dynamical properties related to Shannon-Boltzmann
entropy, Fisher information, as well as Wasserstein metric on
graphs will be presented.

These are based on joint works with Prof. Shui-Nee Chow and Prof.
Haomin Zhou.
Title:
Analysis of ion channel problems

Abstract:
Function of ion channels is a central topic of physiology. It concerns how ions migrate through ion channels that produces electric signals and performs biological tasks. Poisson-Nernst-Planck systems serve as basic models for ionic flow and have shown great successes even in predicting new phenomena.

Based on the modern abstract invariant manifold theory together with the revealing of concrete intrinsic structures of ion channel problems, a geometric singular perturbation framework was developed for analyzing Poisson-Nernst-Planck systems.

The talk will focus on implications of the analysis on properties of ion channels.
Title:
SRB measure and Horseshoe for infinite dimensional dynamical systems.

Abstract:
TBA
Title:
Some Generic Properties of Delay-Differential Equations

Abstract:
We discuss some generic properties of delay-differential equations, in the spirit of the Kupka-Smale Theorem. Results are very dependent on the class of equations considered; for example generic hyperbolicity of periodic solutions holds for equations of the form $\dot{x}(t)=f(x(t),x(t-1))$, but is unknown for equations of the form $\dot{x}(t)=f(x(t-1))$.
This study is motivated in part by efforts to understand global continuation and global bifurcation of periodic orbits for such equations.
Title: How to rigorously solve $\dot{x} = ?$

Abstract: In the setting of nonlinear dynamics associated with multiscale problems, e.g., the life sciences, or data driven science and engineering, it is extremely difficult to derive nonlinearities from first principles and make precise measurements of parameters. The focus of this talk is on how one can make mathematically rigorous statements about the global dynamics of differential equations over large ranges of parameter values without an analytic description of the nonlinearities.
Title:
Transition Fronts in Nonlocal Dispersal Evolution Equations

Abstract:
The current talk is concerned with transition fronts of nonlocal dispersal evolution equations in heterogeneous media. As it is known, solutions of nonlocal dispersal evolution equations do not become smoother in space as time elapses. This lack of space regularity would cause a lot of difficulties in studying transition fronts in nonlocal dispersal evolution equations. In the current talk, I will first present some general criteria concerning space regularity of transition fronts in nonlocal dispersal evolution equations with a large class of nonlinearities. I will then discuss the existence, uniqueness, and stability of transition fronts on nonlocal dispersal evolution equations with monostable, bistable, and ignition type nonlinearities in time heterogeneous media.
Erik S. Van Vleck, University of Kansas

Title:
Projected Data Assimilation and Applications

Abstract:
In this talk we present a framework for a class of DA techniques based upon a computational time dependent slow/fast splitting. We will discuss advantages and disadvantages of such techniques as well classes of problems for which these techniques are well suited. We outline some specific techniques we have been developing and illustrate their effectiveness through computational results. Finally, we investigate the potential of such techniques for improved uncertainty quantification. Time permitting we will present some applications of these results to a single column radiation-convection model and a land-surface model.
James (Jim) Yorke, University of Maryland

Title:

WHAT'S THE POINT???

Abstract:

Revealing fixed points ....
Title:
Dynamics of Tensor Approximation in Narrow Valleys

Abstract:
Approximating a multivariate function/tensor as a short sum of separable functions/tensors has many important potential uses, but common optimization algorithms applied to solve this problem can exhibit extremely slow progress in regions known informally as "swamps". Swamps have hindered the progress of various applications for decades and need to be understood so they can be alleviated. We have identified one possible type of swamp as a narrow valley in the optimization landscape. We analyze the dynamics of one important class of algorithms, Block Coordinate Descent (BCD), in typical valleys and identify several interesting and potentially useful properties. We have also discovered that canonical low rank tensor approximation problems can possess essential singularities from which emanate very narrow valleys. This may explain why swamps are encountered so frequently in practice. We hope that these observations will inform attempts to accelerate the algorithms.

Authors: Martin Mohlenkamp, Balazs Barany, Todd Young
Title:
What does the average eigenvalue know?

Abstract:
The average of the first $k$ eigenvalues of a self-adjoint operator, as a function of $k$, has implications for inverse problems. What information does it contain about an operator, or about the shape of a domain or graph on which the operator is defined? I'll describe some recently developed tools for approaching inverse problems through averages of operators, with selected applications to PDEs on domains and to graphs.

This work is joint with J. Stubbe of EPFL and, in part, A. El Soufi and S.Ilias of the University of Tours.
Title: Path optimization for surveillance

Abstract: We propose a computational strategy to find the optimal path for a mobile sensor with limited coverage to surveillance a given region. The goal is to find one of the shortest feasible paths to achieve the complete scan of the environment. The path length is optimized by reducing its length, via solving a system of ordinary differential equations, while maintaining the complete scan of the environment. Furthermore, we use intermittent diffusion, which converts the ODEs into stochastic differential equations (SDEs), to find an optimal path whose length is globally minimal. This work adapts global optimizations by intermittent diffusion a work by Shui-Nee Chow, Tzi-Sheng Yang and Haomin Zhou, and finding shortest path through region with obstacles by Shui-Nee Chow, Jun Lu and Haomin Zhou.

This is joint work with Seong Jun Kim, Haomin Zhou, and Ben Ide (GT Math).
Abstract:
Stochastic averaging has a long history for systems with multiple
time scales and Gaussian forcing, but far less attention has been
paid to problems where the stochastic forcing has infinite variance,
such as in Levy processes or alpha-stable noise. Correlated
additive and multiplicative (CAM) Gaussian noise, with infinite
variance or ``fat tails'' in certain parameter regimes, can arise
generically in many models with parametric uncertainty and has
received increased attention in the context of atmosphere and
ocean dynamics. These applications motivate new reduced models
using stochastic averaging for systems with fast processes driven
by noise with fat tails. We develop these results for the case of
alpha-stable noise, giving explicit results that use the Marcus
interpretation, the infinite variance analog to the Stratonovich
interpretation. Then we show how reduced models for systems
driven by fast linear CAM noise processes can be connected with
the stochastic averaging for multiple scales systems driven by
alpha-stable processes. We identify the conditions under which
the approximation of a CAM noise process is valid in the averaged
system, and illustrate methods using effectively equivalent fast,
infinite-variance processes. These new types of approximations
open the door for stochastic averaging in a wider range of
stochastic systems with multiple time scales.

This is joint work with Prof. Adam Monahan (U Victoria) and Dr. Will
Thompson (UBC/NMi Metrology and Gaming).
Title:
Manifolds on the verge of a hyperbolicity breakdown

Abstract:
Normal hyperbolicity is an important concept since manifolds with this property persist. We explore numerically the boundary of validity of normal hyperbolicity (This requires very fast and reliable algorithms backed by "a-posteriori" theorems). We found an interesting scenario in which the exponents remain bounded away, but the stable and unstable directions get close. We find several scaling relations. Some of them have been proved recently.

Joint work with Alex Haro.
Title:
Antibiotic Cycling: A cautionary tale

Abstract:
Antibiotics have greatly reduced the morbidity and mortality due to infectious diseases. Although antibiotic resistance is not a new problem, its breadth now constitutes a significant threat to human health. One strategy to help combat resistance is to find novel ways to use existing drugs, even those that display high rates of resistance. For the pathogens Escherichia coli and Pseudomonas aeruginosa, pairs of antibiotics have been identified for which evolution of resistance to drug A increases sensitivity to drug B and visa versa. These research groups have proposed cycling such pairs to treat infections, similar treatment strategies are being investigated for various cancer forms as well.

While an exciting treatment prospect, no cycling experiments have yet been performed with consideration of pharmacokinetics (PK) and pharmacodynamics (PD). To test the plausibility of this scheme and search for ways to optimize it, we create a mathematical model with explicit PK-PD parameters. We study several possible treatment protocols using pairs and triplets of such antibiotics, and investigate the speed of ascent of multiply resistant mutants. Our analyses show that the likelihood of treatment failure due to rapid ascent and fixation of resistant mutants is high.
Abstract:
We consider a general linear Hamiltonian system \( u_t = JL u \) in a Hilbert space \( X \) -- the energy space. The main assumption is that the energy functional \( \frac{1}{2} \langle Lu, u \rangle \) has only finitely many negative dimensions -- \( n^-(L) < \infty \). Our first result is an index theorem related to the linear instability of \( e^{tJL} \), which gives some relationship between \( n^-(L) \) and the dimensions of spaces of generalized eigenvectors of eigenvalues of \( JL \). Under some additional non-degeneracy assumption, for each eigenvalue \( \lambda \in i\mathbb{R} \) of \( JL \) we also construct special ``good'' choice of generalized eigenvectors which both realize the corresponding Jordan canonical form corresponding to \( \lambda \) and work well with \( L \). Our second result is the linear exponential trichotomy of the group \( e^{tJL} \). This includes the nonexistence of exponential growth in the finite co-dimensional invariant center subspace and the optimal bounds on the algebraic growth rate there. Thirdly we consider the structural stability of this type of systems under perturbations if time permits. Finally we discuss applications to examples of nonlinear Hamiltonian PDEs such as BBM, GP, and 2-D Euler equations, including the construction of some local invariant manifolds near some coherent states (standing wave, steady state, traveling waves etc.).