FORMULA SHEET FOR TEST #1, MATH 1502

You may need one or more of the following formulas.

DIVERGENCE (OR NTH TERM) TEST: Given $\sum_{k=1}^{\infty} a_k$, if $\lim_{k\to\infty} a_k \neq 0$, then $\sum_{k=1}^{\infty} a_k$ diverges.

COMPARISON TEST: Let $\sum_{k=1}^{\infty} a_k$ be a series with $a_k \geq 0$ for all k.

(a) If we can find a series $\sum_{k=1}^{\infty} b_k$ such that $\sum_{k=1}^{\infty} b_k$ CONVERGES and $a_k \leq b_k$ for all but finitely many terms, then $\sum_{k=1}^{\infty} a_k$ CONVERGES as well.

(b) If we can find a series $\sum_{k=1}^{\infty} c_k$ such that $\sum_{k=1}^{\infty} c_k$ DIVERGES and $a_k \geq c_k \geq 0$ for all but finitely many terms, then $\sum_{k=1}^{\infty} a_k$ DIVERGES as well.

INTEGRAL TEST: Let f be a continuous, positive, and decreasing function. Then $\sum_{k=1}^{\infty} f(k)$ converges if and only if $\int_1^{\infty} f(x)dx$ converges, and diverges if and only if $\int_1^{N} f(x)dx \to \infty$ as $N \to \infty$.

LIMIT COMPARISON TEST: Let $\sum_{k=1}^{\infty} a_k$ be a series with positive terms. Select a series $\sum_{k=1}^{\infty} b_k$. If $\lim_{k\to\infty} \frac{a_k}{b_k} = c$, where $0 < c < \infty$, then both series converge or both series diverge.

RATIO TEST: Let $\sum_{k=1}^{\infty} a_k$ be a series with all positive terms. Let $L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

(a) If $L < 1$, then $\sum_{k=1}^{\infty} a_k$ converges.

(b) If $L > 1$, then $\sum_{k=1}^{\infty} a_k$ diverges.

ROOT TEST: Let $\sum_{k=1}^{\infty} a_k$ be a series of positive terms. Let $R = \lim_{n\to\infty} (a_n)^{1/n}$.

(a) If $R < 1$, then $\sum_{k=1}^{\infty} a_k$ converges.

(b) If $R > 1$, then $\sum_{k=1}^{\infty} a_k$ diverges.

ALTERNATING SERIES TEST: Let $\sum_{k=1}^{\infty} (-1)^k a_k$ be an alternating series. Then the series converges if the terms are decreasing and $\lim_{n\to\infty} a_n = 0$.

TAYLOR POLYNOMIAL:

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

TAYLOR REMAINDER:

$$R_{n+1}(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

COMMON MACLAURIN SERIES:

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$\ln(1+x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1}$$

TRAPEZOIDAL AND SIMPSON'S RULES:

$$\int_a^b f(x)dx \approx \frac{1}{2}\Delta x[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)]$$

$$\int_a^b f(x)dx \approx \frac{1}{6}\Delta x[f(x_0) + 4f(\frac{x_0+x_1}{2}) + 2f(x_1) + ... + 2f(x_{n-1}) + 4f(\frac{x_{n-1}+x_n}{2}) + f(x_n)]$$

FIRST-ORDER LINEAR DEQ:

$$y = \frac{\int q(x)e^{\int p(x)dx}dx + C}{e^{\int p(x)dx}}$$