Applicant Name: ________________________________

Instructions:
1) If you have not completed a math course past the 1000-level, then complete ALL OF PROBLEMS 1-6.
2) If you have completed at least one 2000-level math course, then WORK EXACTLY SIX OF THE LISTED PROBLEMS, where you have chosen questions from AT LEAST THREE different sections.

SHOW WORK as if you were teaching the problems in your recitation class. Your solutions must be written out by hand. Your handwritten solutions should be turned in to the undergraduate office by the application deadline.

If you are selected for an interview, you will be asked to present one of the problems during your interview. You should present the problem as if you are teaching in front of a live class. As problem sets will not be returned, please keep a copy of your solutions for your record. Please limit your presentation to no more than 10 minutes.

Since TAs have ample resources at their disposal prior to class time, you are allowed to use books and other written resources to help solve these problems. However, since the problem set is a portion of the UTA/ULA application, you are NOT allowed to:

- ask current or past math TAs for help
- ask professors for help
- use Wolfram Alpha, or other internet sources, to solve the problems
- work together with another applicant
- copy directly from ANY source

Please sign the honor statement below that certifies you have submitted your own solutions to the problem set, and submit this page along with your solutions: I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.

Sign Your Name: ________________________________
Problem Set

Differential and Integral Calculus Questions

1. Evaluate the integral
\[ \int \frac{dx}{\sqrt{16 + e^{2x}}}. \]
Show work to justify all steps using techniques of integration. Using an integral table is not sufficient explanation.

2. Using Calculus, sketch a graph of the function
\[ f(x) = \frac{x^3 + 1}{x^2 - 1}. \]

3. Use a MacLaurin series to estimate within an error of at most 0.001:
\[ \int_0^{0.5} \sin(x^2)dx. \]

4. Two ships, A and B, are docked at the same port. At noon, Ship A begins to travel north at a rate of 20 mi/hr. Ship B leaves at 2:00 pm and travels east at a rate of 25 mi/hr. How fast is the distance between the ships changing at 4:00 pm?

Linear Algebra Questions

5. Find \( A^{10} \) if \( A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix} \).

6. For the matrix below, determine: (i) the rank and nullity; and (ii) a basis for the column space and nullspace.
\[ \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix} \]
Multivariable Calculus Questions

7. Find the stationary points and the local extreme values of the function

\[ f(x, y) = \frac{-2x}{x^2 + y^2 + 1}. \]

8. Evaluate the double integral

\[ \int_{\Omega} \int e^{x^2} dxdy, \]

where \( \Omega \) is the triangle formed by the \( x \)-axis, \( 2y = x \), and \( x = 2 \).

Differential Equations Questions

9. Find the solution of the given initial value problem.

\[ t^3 y' + 4t^2 y = e^{-t}, \quad y(-1) = 0, \quad t < 0 \]

10. Find the solution of the given initial value problem.

\[ \vec{x}' = \begin{pmatrix} 1 & -1 \\ 1 & -3 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 5 \\ -8 \end{pmatrix} \]