School of Mathematics  
Georgia Institute of Technology  
Undergraduate TA Application Problem Set  
Fall 2019

Applicant Name:  ____________________________________________

Instructions:
WORK ANY FIVE, AND ONLY FIVE, OF THE SIX LISTED PROBLEMS.

SHOW WORK as if you were teaching the problems in your recitation class. Your solutions must be written out by hand. Your handwritten solutions should be scanned and uploaded to the problem set assignment on the Canvas application site by the application deadline.

If you are selected for an interview, you will be asked to present one of the problems during your interview. You should present the problem as if you are teaching in front of a live class. As problem sets will not be returned, please keep a copy of your solutions for your record. Please limit your presentation to no more than 10 minutes.

Since TAs have ample resources at their disposal prior to classtime, you are allowed to use books and other written resources to help solve these problems. However, since the problem set is a portion of the UTA application, you are NOT allowed to:
– ask current or past math TAs for help
– ask professors for help
– use Wolfram Alpha, or other internet sources, to solve the problems
– work together with another applicant
– copy directly from ANY source

Please sign the honor statement below that certifies you have submitted your own solutions to the problem set, and submit this page along with your solutions: I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.

Sign Your Name:  ____________________________________________
Problem Set

1. At noon, you are running to get to class and notice a friend 100 feet west of you, also running to class. If you are running south at a constant rate of 450 ft/min (approximately 5 mph) and your friend is running north at a constant rate of 350 ft/min (approximately 4 mph), how fast is the distance between you and your friend changing at 12:02 pm?

2. Suppose you are given the following information about the derivative of a function $f(x)$.
   The function $f(x)$ is continuous and differentiable on $(-\infty, \infty)$.
   
   \[
   f'(x) > 0 \text{ on the intervals } (-\infty, 0) \cup (3, 5) \\
   f'(x) < 0 \text{ on the intervals } (0, 3) \cup (5, \infty)
   \]
   
   (a) Does $f(x)$ have an absolute maximum on $(-\infty, \infty)$? (Definitely yes, definitely no, or maybe?) Explain your answer.
   (b) Does $f(x)$ have an absolute minimum on $(-\infty, \infty)$? Explain your answer.
   (c) Does $f(x)$ have an absolute maximum on $[-2, 10]$? If so, where could it be? If not, explain why not.
   (d) Does $f(x)$ have an absolute minimum on $[-2, 10]$? If so, where could it be? If not, explain why not.

3. Consider the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (z, x)$.
   
   (i) Find the standard matrix $A$ for $T$.
   (ii) Is $T$ onto? Justify your answer.
   (iii) Find vectors $x$ and $y$ satisfying $x \neq y$ and $T(x) = T(y)$.

4. Let $A = \begin{bmatrix} 1 & -2 \\ 0 & 2 \end{bmatrix}$. Draw the eigenspaces of $A$. Clearly label each eigenspace.

5. Find the maximum and minimum values of $f(x, y, z) = x + 2y - 3z$ on the sphere $x^2 + y^2 + z^2 = 56$.

6. Calculate $T$, $N$, and $\kappa$ for the curve $r(t) = \cos^3(t)i + \sin^3(t)j + 3k$, where $0 < t < \frac{\pi}{2}$. 