4317

1 K We discuss $E \setminus \bigcap_1^n A_i = \bigcup_1^n E \setminus A_i$, as the other de Morgan Law is similar. The ~~case~~ proof is by induction; and $n = 2$ is the 1st nontrivial case.

$$E \setminus (A_1 \cap A_2) = (E \setminus A_1) \cup (E \setminus A_2).$$

note $x \in E \setminus A_1 \cap A_2$ iff $x \in E$ and $x \notin A_1 \cap A_2$

iff $x \in E$ and not $[x \in A_1$ and $x \in A_2]$

iff $x \in E$ and $(x \notin A_1$ or $x \notin A_2)$

iff $(x \in E$ and $x \notin A_1)$ or $(x \in E$ and $x \notin A_2)$

iff $x \in E \setminus A_1 \cup E \setminus A_2$

In the induction phase assume statement true for n sets, ~~And pro~~ $A_1, —, A_n$. And prove it for $n+1$ sets $A_1, —, A_{n+1}$. Write $A_n' = (A_n \cap A_{n+1})$.

$$E \setminus \bigcap_1^{n+1} A_n = E \setminus \left\{ \bigcap_1^{n-1} A_j \cap A_n' \right\}$$

by case of $n=2$
$$= \left\{ E \setminus \bigcap_1^{n-1} A_j \right\} \cup \left\{ E \setminus A_n' \right\}$$

by induction + $n=2$
$$= \left\{ \bigcup_1^{n-1} E \setminus A_j \right\} \cup \left\{ E \setminus A_n \cup E \setminus A_{n+1} \right\}$$

$$= \bigcup_1^{n+1} E \setminus A_n$$

2 G

. $g \circ f(x) = x \quad \forall \; x \in D(f)$

clearly implies g is defined on the range of f

$R(f) \subseteq D(g)$. And range of g includes

all $x \in D(f)$ so $R(g) \supseteq D(f)$

Suppose $f(x) = f(x')$, $x, x' \in D(f)$. Then

$x = g(f(x)) = g(f(x')) = x'$. So f is an injection.

2 H By G, f is 1-1 on it's domain.

Hence it has an inverse. Using both hypotheses

and G, $R(f) \subseteq D(g) \subseteq R(f)$

and simarly, $R(g) = D(f)$, so g is indeed f^{-1}!

3 F The basic example is the 'shift'

$\sigma : \mathbb{N} \to \mathbb{N}$ given by $\sigma(n) = n+1$. So

no point is mapped onto 1.

If A is infinite let $B = \{b_1, b_2, \ldots\}$ be c'tble

in A. Define $\varphi(x) = x$, $x \notin B$, &

$\varphi(b_n) = b_{n+1}$ Then φ is 1-1, & no point

is mapped onto b_1.

3 J Let $\varphi : \{1, \dots, N\}$ be 1-1. Then the range of φ must ~~be~~ not contain all integers $\{1, \dots N+1\}$. For if it did, we would have $N+1 \leq N$. Thus there is some $1 \leq j \leq N+1$ not in the range of φ.

$\mathbb{N} \times \mathbb{N}$ is denumerable

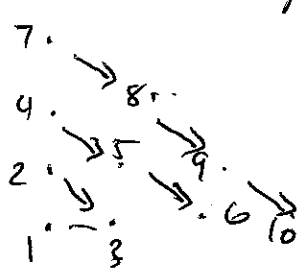
It suffices to construct an injection φ from $\mathbb{N} \times \mathbb{N}$ into $\mathbb{N}$. Consider

$$\varphi(m, n) = 2^m \cdot 3^n.$$

~~Also~~ By the uniqueness of prime factorization $\varphi(m, n)$ is 1-1: If $\varphi(m,n) = \varphi(m', n')$ we necessarily must have $m = m'$ & $n = n'$.

Honor & dignity require a bijection. I leave it to you to verify that one is given by

$$\varphi(m,n) = \frac{(m+n)(m+n-1)}{2} + m - 1$$



1 K We discuss $E \cup \bigcap_1^n A_i = \bigcap_1^n E \cup A_i$

as the other ~~de~~ de Morgan Law is similar.

Use induction. 1st non-trivial case is $n = 2$:

$$E \cup (A_1 \cap A_2) = \cap (E$$

$$\frac{(m+n)(m+n-1)}{2} + m - 1$$

$\varphi(1,1) = 1$

$\varphi(1,2) = 2$

$\varphi(2,1) = 3$