Minimization of Electrostatic Potential of Charged Closed Curves

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Project Description

In this project, we suppose that an arbitrary simply connected closed polygonal curve (lying in $\mathbb{R}^2$) has electric charges distributed uniformly about its perimeter. Eventually, we would like to remove the restriction of the curve being a polygon, and consider any continuous closed curve. The objective is then to modify the curve (i.e., stretch, translate, rotate its segments) so as to minimize its electrostatic potential, keeping the curve a polygon. This will have application in an image-driven evolution to ensure that any simply connected polygon will never intersect itself. The method we shall employ is to derive equations for the potential between two charged segments as shown in Figure 1, and then to apply superposition to find the potential of the entire polygon. We can then find the gradient of the potential and then apply a gradient descent algorithm.

During the summer, we will derive all necessary equations for the polygon case, and implement the algorithm on a computer. We will also consider the case when the curve is not restricted to being a polygon. Intuitively, we would expect a polygon subject to the charge distribution to become regular, and any continuous closed curve (without the constraint of being a particular shape) to become a circle. We would like to prove this if possible.

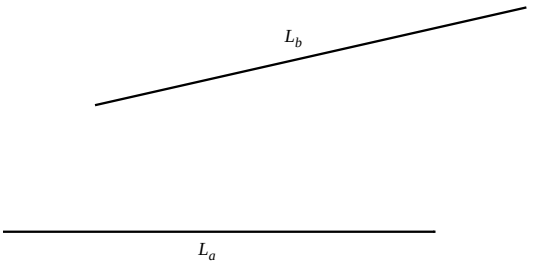

Figure 1: Two disconnected charged segments.