1. Show that every $n$-dimensional polytope in $\mathbb{R}^n$ has at least $n + 1$ facets.

2. Let $P$ be the polyhedral set consisting of all points in $\mathbb{R}^2$ satisfying the inequalities $x \geq 0, -x + y + 1 \geq 0, x + 3y - 1 \geq 0, 3x + y - 1 \geq 0$. Express $P$ as a vector sum of a polytope and a finitely generated convex cone.

3. Suppose $T$ is a simplicial complex, and $B(T)$ is its barycentric subdivision. Show that $T$ and $B(T)$ have the same Euler characteristic.

4. Suppose $T$ is a simplicial complex, and $T^{(1)}$ its one-dimensional skeleton, i.e. the subcollection of all simplexes of dimension $\leq 1$. Show that $|T|$ is connected if and only if $|T^{(1)}|$ is.

5. Suppose the maps $f, g : X \to Y$ are homotopic, and $h : Y \to Z$. (All the maps are continuous). Show that $h \circ f$ and $h \circ g$ are homotopic.

6. Suppose $f : X \to S^n$ is a continuous map which is not surjective. Show that $f$ is homotopic to a constant map, i.e. a map $g : X \to S^n$ such that $f(x) = f(y)$ for every $x, y \in X$.

7. Show that the diameter of a simplex is the length of its longest edge. More generally, the diameter $\text{Conv}(A)$ is equal to the diameter of $A$. 