Chapter II, Section 2

Problem 1:

f has no critical points $f = 2 - x - 2y$

$$\nabla f = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.$$

Boundary of D: $g = x^2 + 4y^2 + 3y - 8 = 0$

$$\nabla g = \begin{bmatrix} 2x \\ 8y+3 \end{bmatrix}.$$

$\nabla f \parallel \nabla g \Rightarrow (8y+3)\cdot 1 - 4x = 0$

$-4x + 8y + 3 = 0$ $\qquad$ $x^2 + 4y^2 + 3y = 8$

$\Rightarrow x = 2y + \frac{3}{4}$

$$\left(2y + \frac{3}{4}\right)^2 + 4y^2 + 3y = 8$$

$$8y^2 + 6y = 8 - \frac{9}{16} \text{ or}$$

$$y^2 + \frac{3}{4}y = 1 - \frac{9}{8\cdot 16}$$

$$\left(y + \frac{3}{8}\right)^2 = 1 - \frac{9}{8\cdot 16} + \frac{9}{64} = \frac{8\cdot 16 - 9 + 9\cdot 2}{8\cdot 16}$$

$$y_{1,2} = \frac{3}{2} \pm \sqrt{\frac{137}{128}} \qquad x_{1,2} = \frac{15}{4} \pm 2\sqrt{\frac{137}{128}}$$

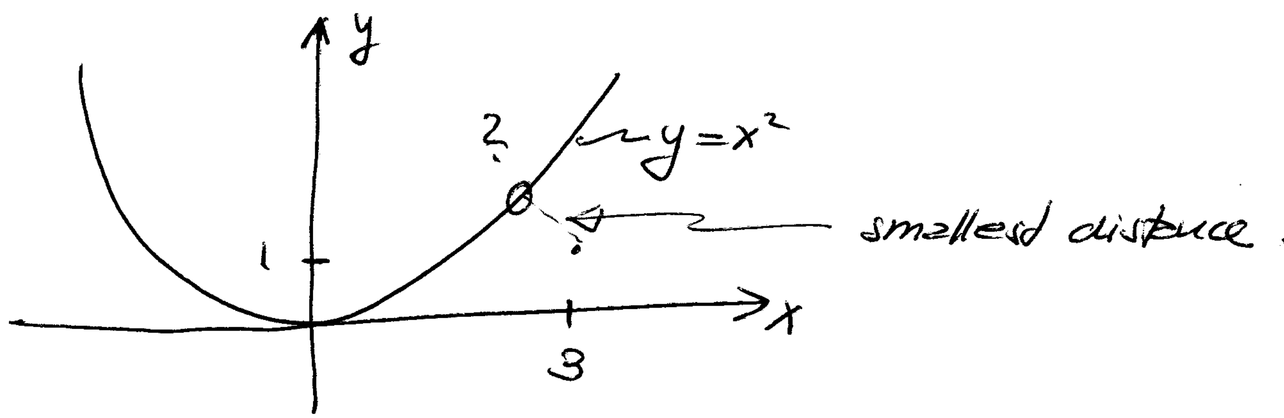
f has its max value at (x_2, y_2)

$$= \cancel{\text{at}} \quad -\frac{19}{4} + 4\sqrt{\frac{137}{128}}$$

min. value:

$$-\frac{19}{4} - 4\sqrt{\frac{137}{128}}$$

Problem 3:



Minimize $(x-3)^2 + (y-1)^2$ subject to the constraint that $y = x^2$.

$f(x,y) = (x-3)^2 + (y-1)^2 \quad g(x,y) = x^2 - y$

Constraint: $g(x,y) = 0$.

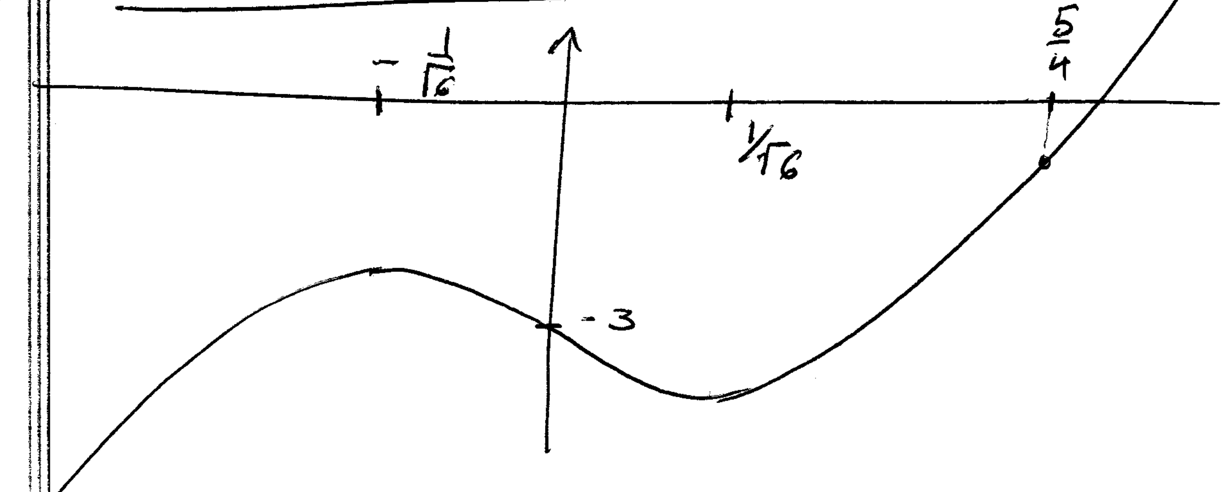
$$\nabla f = 2\begin{bmatrix} x-3 \\ y-1 \end{bmatrix} \quad \nabla g = \begin{bmatrix} 2x \\ -1 \end{bmatrix}$$

Points where $\nabla f \parallel \nabla g$:

$$(-1)(x-3) - (y-1)2x = 0$$

or $\quad 2yx - x - 3 = 0, \quad x^2 = y$

$h(x) = 2x^3 - x - 3 = 0$



$h(\frac{5}{4}) = -\frac{11}{32}$.

Apply Newton with starting point $5/4$.

$$x_1 = x_0 - \frac{h(x_0)}{h'(x_0)} = \frac{5}{4} + \frac{11}{3}\,\frac{1}{6(\frac{5}{4})^2 - 1} = \frac{173}{134}$$

$h(x_1) \simeq 0.013$

We use now $x_1 = \frac{173}{137}$ as an approximate value for the root.

$y_1 = x_1^2 \qquad$ Distance: $\sqrt{(x_1 - 3)^2 + (y_1 - 1)^2}$

$\simeq 1.894$