

Practice Test 3 B

Problem 1: For the matrix

$$A = \begin{bmatrix} 1 & -5 \\ 2 & -1 \end{bmatrix}$$

- Find the QR factorization using Householder reflections.
- Find the Schur factorization.
- Compute e^{At} .

Problem 2: Consider the curve

$$x(t) = 1 - \cos(t) , \quad y(t) = t - \sin(t) , \quad 0 < t < 2\pi .$$

- Find the velocity $\mathbf{v}(t)$ and the acceleration $\mathbf{a}(t)$.
- Compute the unit tangent - and the unit normal vector to the curve.
- Find the length of this curve.
- Compute the curvature $\kappa(t)$.

Problem 3: Consider the rotation matrix

$$Q = \frac{1}{14} \begin{bmatrix} 6 & -4 & -12 \\ 12 & 6 & 4 \\ 4 & -12 & 6 \end{bmatrix} .$$

- Find the angle of rotation.
- Find the axis of rotation.
- Find a square root of the matrix Q .

Problem 4: Given the differential equation $x'' + \mu(x^2 - 1) + x = 0$.

- Write this differential equation as a first order system.
- Find all the critical points.
- Linearize the system around these critical points.
- Draw a few curves for the phase portrait for the linearized system. How does it look for various values of μ ?
- Determine the stability of the critical points for various values of μ .