Pratice Test 1B for Math2605, Spring 2004

Problem 1

a) Calculate the gradient of the function

$$f(x, y, z) = x^4 + y^4 + z^4 + xyz$$

at the point $(1, 2, 1)$.

b) Find the linear approximation of the function $f(x, yz)$ at the point $(1, 2, 1)$.

c) Calculate the the equation for the plane tangent to the surface $f(x, y, z) = f(1, 2, 1)$ at the point $(1, 2, 1)$.

c) Find any two linearly independent vectors that are tangent to the surface $f(x, y, z) = f(1, 2, 1)$ at the point $(1, 2, 1)$.

Problem 2 Given two functions

$$f(x, y) = x^3 - 2xy \ , g(x, y) = x^3 - y^3 \ .$$

a) Find the intersection of the curves $f(x, y) = -1$ and $g(x, y) = 0$.

b) Find the lines tangent to the curves $f(x, y) = -1$ and $g(x, y) = 0$.

c) Calculate the angle between these two tangent lines.

Problem 3 Consider the two function $f(x, y, z) = x^2 - y^2$ and the function $g(x, y, z) = x^4 + y^4 + z^4$. The level sets $f = 1$ and $g = 2$ intersect in some curve. Note that the point $(1, 0, 1)$ is on that curve.

a) Calculate the gradient of the two functions at the point $(1, 0, 1)$.

b) Find the line tangent to both level sets at the point $(1, 0, 1)$.

Extra credit:

Find the distance between the curve $x^2 - y^2 = 4$ and the line $y = 2x$.