Problem 1
For the matrix
\[ A = \begin{bmatrix} 4 & 8 \\ 3 & 2 \end{bmatrix} \]
a) Find the QR factorization of A.
b) The Schur factorization.
c) Compute \( e^{At} \).

Problem 2 Consider the rotation matrix
\[ Q = \frac{1}{45} \begin{bmatrix} 40 & -5 & 20 \\ 13 & 40 & -16 \\ -16 & 20 & 37 \end{bmatrix} \]
a) Find the axis of rotation \( u \) and the angle of rotation \( \theta \).
b) Find \( B \) so that \( Q = e^{\theta B} \).
c) Find a rotation \( R \) so that \( Q = R^2 \).
d) Find the family of rotations that interpolate the identity and \( Q \).

Problem 3 Consider the differential equation
\[ x'' = x' - x + x^3. \]
a) Write this equation as a first order system.
b) Find all the critical points.
c) Find the type of the critical points and decide whether they are linearly stable or unstable.
d) Decide which ones are stable or unstable for the nonlinear system and determine their possible types.

Problem 4 Consider the curve
\[ x(t) = \frac{1}{2} t^2 - \frac{1}{3} t^3, \quad y(t) = \frac{1}{2} t^2 + \frac{1}{3} t^3, \quad 1 \leq t \leq 2. \]
a) Find the velocity for all values of \( t \).
b) Find the unit tangent vector \( T(t) \).
c) Find the length of the curve.
d) Rewrite the curve in terms of the length parametrization \( s \).
e) Find the curvature \( \kappa(s) \).