**Solutions of Quiz 2b for Calculus ++, Math 2605 J1-2, September 11, 2007**

**Name:**

This quiz is to be taken without calculators and notes of any sorts. The allowed time is 20 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

Consider the function $f(x,y) = xy + x^3$.

**I:** (3 points) Find the equation of the plane that is tangent to the graph of $f$ at the point $(1,2)$.

$$\nabla f(x,y) = \begin{bmatrix} y+3x^2 \\ x \end{bmatrix} \nabla f(1,2) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$z = 3 + 5(x-1) + (y-2)$$

**II:** (3 points) Find the line that is tangent to the level curve of the function $f$ at the point $(1,2)$. Give the line in parametrized form.

Direction vector is

$$\begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

which is perpendicular to $\nabla f(1,2)$. The line is therefore given by

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} -1 \\ 5 \end{bmatrix} .$$

**III:** (2 points) Find the rate of change of the function $f(x,y)$ at the point $(1,2)$ in the direction $(2,2)$.

$$\begin{bmatrix} 5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 12 .$$

**IV:** (2 points) Find all the critical points of the function $f(x,y)$.

$$\nabla f(x,y) = \begin{bmatrix} y+3x^2 \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$y + 3x^2 = 0 \ , \ x = 0$$

$(0,0)$ is the only solution.

**Extra credit:** (3 points) Find the curvature of the function $f$, i.e., the second derivative at $t=0$ of the function $g(t) = f(\mathbf{x_0} + t\mathbf{v})$ where $\mathbf{x_0} = (1,2)$ and $\mathbf{v} = (2,1)$.

The Hessian is

$$H_f(x,y) = \begin{bmatrix} 6x & 1 \\ 1 & 0 \end{bmatrix}$$

$$H_f(1,2) = \begin{bmatrix} 6 & 1 \\ 1 & 0 \end{bmatrix}$$

and

$$g''(0) = 28 .$$