Solutions for Quiz 2 for Calculus ++, Math 2605 J1-2, September 11, 2007

Name:

This quiz is to be taken without calculators and notes of any sorts. The allowed time is 20 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

Consider the function $f(x, y) = xy + y^3$.

I: (3 points) Find the equation of the plane that is tangent to the graph of f at the point $(2, 1)$.

$$\nabla f(x, y) = \begin{bmatrix} y \\ x + 3y^2 \end{bmatrix} \quad \nabla f(1, 2) = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$z = 3 + (x - 2) + 5(y - 1)$$

II: (3 points) Find the line that is tangent to the level curve of the function f at the point $(2, 1)$.

Direction vector is

$$\begin{bmatrix} -5 \\ 1 \end{bmatrix}$$

which is perpendicular to $\nabla f(1, 2)$. The line is therefore given by

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} -5 \\ 1 \end{bmatrix} \ .$$

III: (2 points) Find the rate of change of the function $f(x, y)$ at the point $(2, 1)$ in the direction $(1, 2)$.

$$\begin{bmatrix} 1 \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = 12 \ .$$

IV: (2 points) Find all the critical points of the function $f(x, y)$.

$$\nabla f(x, y) = \begin{bmatrix} y \\ x + 3y^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x + 3y^2 = 0 \ , \ y = 0$$

$(0, 0)$ is the only solution.

Extra credit: (3 points) Find the curvature of the function f, i.e., the second derivative at $t = 0$ of the function $g(t) = f(\mathbf{x_0} + t\mathbf{v})$ where $\mathbf{x_0} = (2, 1)$ and $\mathbf{v} = (1, 2)$.

The Hessian is

$$H_f(x, y) = \begin{bmatrix} 0 & 1 \\ 1 & 6y \end{bmatrix}$$

$$H_f(2, 1) = \begin{bmatrix} 0 & 1 \\ 1 & 6 \end{bmatrix}$$

and

$$g''(0) = 28 \ .$$