**Quiz 3 for Calculus ++, Math 2605 J1-2, October 11, 2007**

**Name:**

This quiz is to be taken without calculators and notes of any sorts. The allowed time is 20 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

**I:** (3 points) Using Lagrange multipliers, maximize the function $f(x, y) = x + y$ under the constraint that $x^2 + 2y^2 = 1$.

**II:** (4 points) Calculate the Givens matrix $G$ for the first step in the Jacobi Algorithm for the matrix $A = \begin{bmatrix} 3 & 4 & 2 \\ 4 & -3 & 1 \\ 2 & 1 & 7 \end{bmatrix}$ You do not have to calculate $GAG^T$.

**III:** (3 points) Let $A$ be a symmetric matrix and denote by $A^{(k)}$ the $k$-th Jacobi iterate. If $\mathrm{Off}(A) = 1$ and $\mathrm{Off}(A^{(k)}) \leq \left(\frac{2}{3}\right) \mathrm{Off}(A^{(k-1)})$ , $k = 1, 2, \ldots$ , estimate how many iterations it takes until $\mathrm{Off}(A^{(k)}) \leq 10^{-5}$?

**Extra credit:** (4 points) Consider the matrix $\begin{bmatrix} 2 & 4 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} + t \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ Calculate the eigenvalues $\mu_i(t), i = 1, 2, 3$ for small values of $t$, i.e. calculate $\mu_i(0) + \mu_i'(0)t$.