

Test 1 for Calculus III for CS Majors, Math 2506 J1-J2, September 25, 2007

Name:

This test is to be taken without calculators and notes of any sort. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414... State your work clearly, otherwise credit cannot be given.

Problem 1: Consider the function

$$f(x, y) = x^3 + y^3 - 3xy .$$

a) (5 points) Calculate the gradient at the point $(1, -1)$.

b) (5 points) Find the line (in parametrized form) that is tangent to the curve $f(x, y) = f(1, -1)$ at the point $(1, -1)$.

c) (5 points) Find the best linear approximation of the function $f(x, y)$ at the point $(1, -1)$.

d) (10 points) Find the points on the curve $f(x, y) = f(1, -1)$ where the tangent line is horizontal.

Problem 2: a) (10 points) Calculate the critical points of the function

$$f(x, y) = x^3 + yx^2 - \frac{1}{2}x^2 - y .$$

b) (10 points) Calculate the Hessian at these critical points.

c) (10 points) What are the type of these critical points, are they a max a min or a saddle?

Problem 3: A function $g(x, y)$ has $(0, 0)$ as a critical point and the Hessian at this point is given by

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} .$$

a) (5 points) Write the quadratic approximation $q(x, y)$ for the function $g(x, y)$ in the vicinity of this critical point.

b) (10 points) Find the eigenvalues and the eigenvectors of the Hessian.

c) (10 points) Draw in a qualitative fashion a few of the level curves of $q(x, y)$.

Problem 4: (10 points) a) Set up Newton's scheme for solving the equation $x^2 - y = 1$ and $xy = 1$.

b) (10 points) Use as an initial guess the point $\mathbf{x}_0 = (1, 1)$ and calculate the next approximation \mathbf{x}_1 . Check whether this leads to an improvement.

Extra Credit: (15 points) Given the function $f(x, y) = x^3 - 3xy^2$ and $g(x, y) = 3x^2y - y^3$. What can you say about the angles between lines tangent to the level curves of f resp. g at any point (x, y) ?