First Homework, due Monday September 14, 2009

Please solve problems 1, 4, 5, 8 on page 85-87 of L.C.Evans’ book.

**Problem 5:** Let $U \subset \mathbb{R}^3$ be open and $u(x) \in C^2(U)$ satisfy the equation

$$\Delta u - \mu^2 u = 0,$$

where $\mu > 0$. Show that for any ball $B_r(x) \subset U$

$$u(x) = \frac{\mu r}{\sinh(\mu r)} \int_{\partial B_r(x)} u(y) dS(y).$$

Hint: Show that

$$\text{div} \left[ \frac{\sinh(\mu |x|)}{\mu |x|} \nabla u(x) - u(x) \nabla \frac{\sinh(\mu |x|)}{\mu |x|} \right] = 0$$

and integrate this identity over the ball $B_r(x)$. 