Chapter II, Section 1

Problem 1

a) $$J_{\vec F}(\vec x) = \begin{bmatrix} 3x^2+y & x \\ -2x & -8y \end{bmatrix} \qquad J_{\vec F}(\vec x_0) = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$$

b) Goal: Solve $\vec F(\vec x) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

One iteration:

$$\vec x_1 = \vec x_0 - J_{\vec F}(\vec x_0)^{-1}\,\vec F(\vec x_0)$$

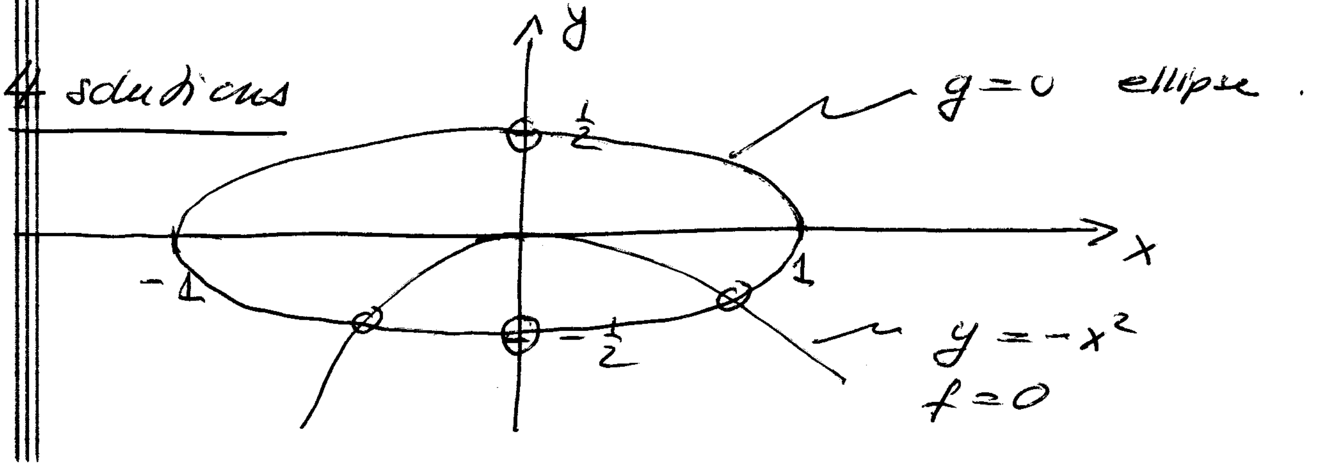
$$J_{\vec F}(\vec x_0)^{-1} = \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\vec F(\vec x_0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad J_{\vec F}(\vec x_0)^{-1}\vec F(\vec x_0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\vec x_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

c) $\vec F(1,-1) = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$, worse than the starting value.

d) 4 solutions



Problem 3

a) $$J_{\vec{F}}(\vec{x}) = \begin{bmatrix} y\cos(xy)-1 & x\cos(xy) \\ 2xy & x^2 \end{bmatrix}$$

$$J_{\vec{F}}(\vec{x}_0) = \begin{bmatrix} \cos(1)-1 & \cos(1) \\ 2 & 1 \end{bmatrix}$$

b) $$\vec{x}_1 = \vec{x}_0 - J_{\vec{F}}(\vec{x}_0)^{-1}\,\vec{F}(\vec{x}_0)$$

$$\vec{F}(\vec{x}_0) = \begin{bmatrix} \sin(1)-1 \\ 0 \end{bmatrix}$$

$$J_{\vec{F}}(\vec{x}_0)^{-1} = \begin{bmatrix} -1 & \cos(1) \\ 2 & 1-\cos(1) \end{bmatrix} \cdot \frac{1}{1+\cos(1)}$$

$$\vec{x}_1 = \begin{bmatrix} \dfrac{\cos(1)+\sin(1)}{1+\cos(1)} \\ \dfrac{3+\cos(1)-2\sin(1)}{1+\cos(1)} \end{bmatrix} \simeq \begin{bmatrix} 0.894 \\ 1.206 \end{bmatrix}$$

c) $$\vec{F}(\vec{x}_1) \approx \begin{bmatrix} -0.013 \\ -0.036 \end{bmatrix} \qquad |\vec{F}(\vec{x}_1)| \approx 0.038$$

$$\vec{F}(\vec{x}_0) \approx \begin{bmatrix} -0.159 \\ 0 \end{bmatrix} \qquad |\vec{F}(\vec{x}_0)| \approx 0.159$$

<u>Improvement.</u>

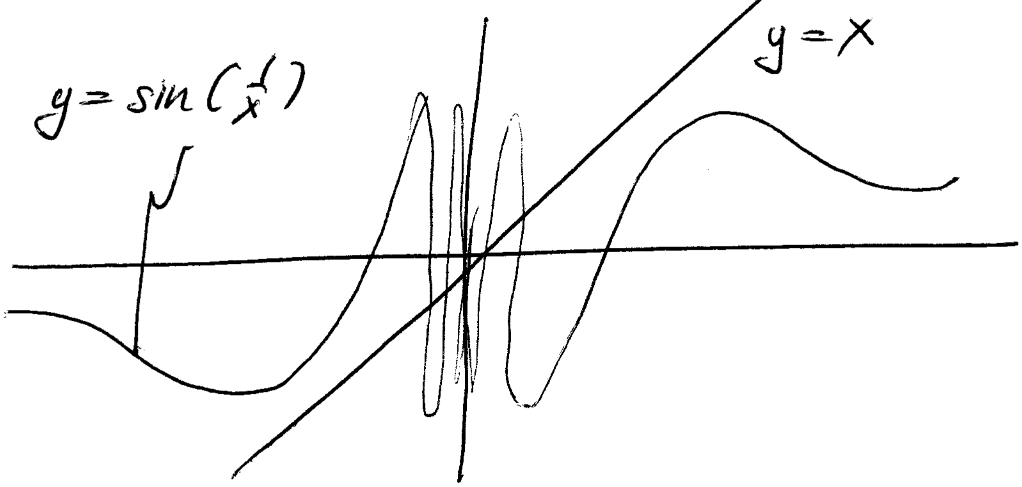
d) Form $g = 0 \Rightarrow x^2 y = 1$ or

$y = \frac{1}{x^2}$

$f = \sin(xy) - x = 0$

$\Rightarrow \sin\left(\frac{1}{x}\right) = x$.

$\sin\left(\frac{1}{x}\right)$ oscillates indefinitely often as x ranges from -2, to 2.



There will be an infinite number of intersection points and hence an infinity of solutions for x and hence for y.

It is hard to calculate all of them with 10 figure accuracy!