Solutions for selected problems in Chapter 0

Please note that there are many solutions for various of the problems. Most of the time you can check by plugging in the given vectors in your answer.

Problem 1.1

a)

$$\mathbf{x}(s,t) = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} .$$

b)

$$\mathbf{z}(u) = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} + u \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix} .$$

c)

$$3x + z = 5 .$$

d)

$$2x + z = 5 \ , 4y + 3z = 5 .$$

e)

$$\begin{bmatrix} 0 \\ -5/2 \\ 5 \end{bmatrix}$$

f) If the interesection point is inside the triangle then, necessarily, we must have

$$\begin{bmatrix} 0 \\ -5/2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + p \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + q \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix} ,$$

where p and q are positive numbers. Solving for p one sees that $p = -1$. Hence the intersection point is not inside the triangle.

g)

$$\mathbf{p_1} - \mathbf{z_0} = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix} ,$$

$$\mathbf{u} = \frac{1}{\sqrt{29}} \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix}$$

$$(\mathbf{p_1} - \mathbf{z_0})_\perp = (\mathbf{p_1} - \mathbf{z_0}) - ((\mathbf{p_1} - \mathbf{z_0}) \cdot \mathbf{u})\mathbf{u} = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix} - \frac{19}{29} \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix} .$$

The distance is therefore

$$|(\mathbf{p_1} - \mathbf{z_0})_\perp| = \frac{\sqrt{1305}}{29} = \frac{\sqrt{5 \cdot 3 \cdot 3 \cdot 29}}{29} = 3\sqrt{\frac{5}{29}} .$$

h)

$$\mathbf{z_0} - \mathbf{p_1} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$$

$$\mathbf{u} = \frac{1}{\sqrt{10}} \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$(\mathbf{z_0} - \mathbf{p_1})_{||} = ((\mathbf{z_0} - \mathbf{p_1}) \cdot \mathbf{u})\mathbf{u}$$

and the distance, therefore, is

$$|(\mathbf{z_0} - \mathbf{p_1}) \cdot \mathbf{u}| = \frac{3}{\sqrt{10}} .$$

Problem 1.5

$$\mathbf{x_0} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

is a base point.

$$\mathbf{p} - \mathbf{x_0} = \begin{bmatrix} -4 \\ -5 \\ 1 \end{bmatrix}$$

$$\mathbf{u} = \frac{1}{\sqrt{11}} \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$$

and the distance is

$$\frac{12}{\sqrt{11}} .$$

Problem 1. 7

There is a misprint in the formulation of the problem: It should read ‘distance from the line’ instead of ‘distance from the plane’.

The intersection of the two planes is given by

$$\mathbf{z}(u) = \frac{1}{2}\begin{bmatrix} 13 \\ 3 \\ 0 \end{bmatrix} + \frac{u}{2}\begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}$$

Try to choose the base point $\mathbf{x_0}$ and parametrization in a simple form, e.g.,

$$\mathbf{x}(s) = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} + t\begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}$$

describes the same line.

$$\mathbf{p} - \mathbf{x_0} = -\begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}, \mathbf{u} = \frac{1}{\sqrt{30}}\begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}.$$

$$(\mathbf{p} - \mathbf{x_0})_\perp = (\mathbf{p} - \mathbf{x_0}) - ((\mathbf{p} - \mathbf{x_0}) \cdot \mathbf{u})\mathbf{u} = \frac{-12}{5}\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

The distance is given by

$$|(\mathbf{p} - \mathbf{x_0})_\perp| = \frac{12}{\sqrt{5}}.$$

Problem 1.9

The line is given by

$$\mathbf{x}(s) = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} + t\begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}.$$

Intersection point with the plane:

$$\frac{1}{9}\begin{bmatrix} 16 \\ 5 \\ 17 \end{bmatrix}.$$

Unit vector, normal to the plane:

$$\frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Reflected line:

$$\mathbf{z}(u) = \frac{1}{9}\begin{bmatrix} 16 \\ 5 \\ 17 \end{bmatrix} + u\begin{bmatrix} -2 \\ 5 \\ -1 \end{bmatrix}.$$

Problem 1.11

$$\mathbf{a_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} , \mathbf{a_2} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} .$$

Problem 1 in section 2:

The problem is to minimize the function

$$f(s,t) = (s-2t)^2 + (3-t)^2 + (1-s-t)^2$$

The minimum is attained at the values

$$s = 10/11 , t = 9/11 .$$

Distance is

$$\frac{8}{\sqrt{11}} .$$

Problem 3 in section 2:

Distance

$$\frac{2}{5}\sqrt{362} .$$