Solution for selected problems from chapter 4

Section 1:

Problem 1

$$\begin{bmatrix} -3 & 4 \\ -6 & 7 \end{bmatrix} = \frac{1}{\sqrt{13}} \begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 10 \\ 0 & 1 \end{bmatrix} \frac{1}{\sqrt{13}} \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$$

Problem 3

$$\begin{bmatrix} & \\ & \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Section 2:

Problem 1

$$|\mathbf{z}| = 4\sqrt{2}$$

Problem 3

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} , \mathbf{z}^{\perp} = \begin{bmatrix} -z_2^* \\ z_1^* \end{bmatrix}$$

$$|\mathbf{z}|^2 = |z_1|^2 + |z_2|^2 = |\mathbf{z}^{\perp}|^2 \ .$$

$$\langle \mathbf{z}, \mathbf{z}^{\perp} \rangle = z_1^*(-z_2^*) + z_2^*(z_1^*) = 0 \ .$$

Problem 5

$$\langle \mathbf{w}, \mathbf{z} \rangle = 9 - 29i$$

Problem 7

$$w = \begin{bmatrix} 4i \\ 2 - 4i \end{bmatrix} \ .$$

The unitary matrix is

$$\frac{1}{6} \begin{bmatrix} 2 + 4i & 4i \\ 4i & 2 - 4i \end{bmatrix} \ .$$

Problem 9: The adjoint is

$$A^* = \begin{bmatrix} 3 - 2i & 1 + 7i \\ 5 - 2i & 2 + i \end{bmatrix} \ .$$

Section 3:

Problem 1

The reflection matrix that sends $\mathbf{x}$ to $5\sqrt{5}\mathbf{e_1}$: $M = I - 2\mathbf{u}\mathbf{u}^t$ where

$$\mathbf{u} = \frac{1}{\sqrt{250 - 110\sqrt{5}}}\begin{bmatrix} 11 - 5\sqrt{5} \\ 2 \end{bmatrix}$$

The vector $\mathbf{u}$ for the reflection that sends $\mathbf{x}$ to $-5\sqrt{5}\mathbf{e_1}$ is given by

$$\frac{1}{\sqrt{250 + 110\sqrt{5}}}\begin{bmatrix} 11 + 5\sqrt{5} \\ 2 \end{bmatrix}$$

Problem 3
The reflection matrix that send $\mathbf{x}$ to $8\mathbf{e_1}$: The vector $\mathbf{u}$ is given by

$$\mathbf{u} = \frac{1}{4\sqrt{3}}\begin{bmatrix} -3 \\ 1 \\ 2 \\ 3 \\ 5 \end{bmatrix}$$

and $M = I - 2\mathbf{u}\mathbf{u}^t$ which, in more detail is given by

$$I - \frac{1}{24}\begin{bmatrix} 9 & -3 & -6 & -9 & -15 \\ -3 & 1 & 2 & 3 & 5 \\ -6 & 2 & 4 & 6 & 10 \\ -9 & 3 & 6 & 9 & 15 \\ -15 & 5 & 10 & 15 & 25 \end{bmatrix}$$

The reflection that maps $\mathbf{x}$ to $-8\mathbf{e_1}$: The vector $\mathbf{u}$ is given by

$$\mathbf{u} = \frac{1}{4\sqrt{13}}\begin{bmatrix} 13 \\ 1 \\ 2 \\ 3 \\ 5 \end{bmatrix},$$

and $M = I - 2\mathbf{u}\mathbf{u}^t$.

Problem 5 The reflection

$$M_1 = \frac{1}{10}\begin{bmatrix} 6 & 0 & 8 \\ 0 & 10 & 0 \\ 8 & 0 & -6 \end{bmatrix}$$

yields

$$M_1 A = \frac{1}{10}\begin{bmatrix} 50 & 50 & 26 \\ 0 & 10 & 10 \\ 0 & 0 & 18 \end{bmatrix}$$

hence the QR factorization is

$$A = \frac{1}{10}\begin{bmatrix} 6 & 0 & 8 \\ 0 & 10 & 0 \\ 8 & 0 & -6 \end{bmatrix} \frac{1}{10}\begin{bmatrix} 50 & 50 & 26 \\ 0 & 10 & 10 \\ 0 & 0 & 18 \end{bmatrix} .$$

Problem 7 The length of the vector $\mathbf{z}$ is $4\sqrt{2}$. Further, the vector

$$\mathbf{w} = -4(1+i)\mathbf{e_1}$$

has the same length as $\mathbf{z}$ and the inner product of $\langle \mathbf{w}, \mathbf{z} \rangle = -8$ which is real. The vector $\mathbf{u}$ is now

$$\mathbf{u} = \frac{1}{4\sqrt{5}}\begin{bmatrix} 5(1+i) \\ -3+2i \\ 1-4i \end{bmatrix}$$

and

$$M = I - \frac{1}{40}\begin{bmatrix} 5(1+i) \\ -3+2i \\ 1-4i \end{bmatrix} [\, 5(1-i), \quad -3-2i, \quad 1+4i \,] \ .$$