Soltions of selected problems in Chapter 6

Section 1:

Problem 1: Intersection points: $(0,0)$ and $(4,-2)$. Slicing in the y direction yields the iterated integral

$$\int_0^4 \left(\int_{-\sqrt{x}}^{-x/2} x^3 y dy \right) dx \ .$$

Slicing in the x direction yields the integral

$$\int_{-2}^0 \left(\int_{y^2}^{-2y} x^3 y dx \right) dy \ .$$

Evaluating the integrals yields: $-256/15$.

Problem 3: The parabola intersects the x axis at the points $(0,0)$ and $(4,0)$. Slicing in the y direction yields the integral

$$\int_0^4 \left(\int_0^{4-(x-2)^2} x^2 y^2 dy \right) dx$$

and slicing in the x-direction yields the integral

$$\int_0^4 \left(\int_{2-\sqrt{4-y}}^{2+\sqrt{4-y}} x^2 y^2 dx \right) dy \ .$$

Evaluating the integrals yields $\frac{2^{15}}{3\cdot 7\cdot 9}$.

Problem 5: The region Ω forms a triangle with vertices $(0,0)$, $(5/3,5/3)$ and $(5,-5)$. Slicing in the y direction leads to two interated integrals:

$$\int_0^{5/3} \left(\int_{-x}^{x} (x^2+y^2) dy \right) dx + \int_{5/3}^{5} \left(\int_{-x}^{5-2x} (x^2+y^2) dy \right) dx \ .$$

Slicing in the x direction leads again to two iterated integrals

$$\int_{-5}^0 \left(\int_{-y}^{(5-y)/2} (x^2+y^2) dx \right) dy + \int_0^{5/3} \left(\int_{y}^{(5-y)/2} (x^2+y^2) dx \right) dy \ .$$

Evaluating the integrals yields $2(5/3)^5$.

Section 2:

Problem 1: The integral is the same as integrating the function $y+1$ over the domain Σ that is inside both of the circles $(x-1)^2+y^2=2$ and $(x+1)^2+y^2$. Since the domain is symmetric in the y-direction $\int_\Sigma y dx dy = 0$. Thus the integral is just the area of the domain Σ which is given by

$$4\int_0^{\sqrt{2}-1} \sqrt{2-(x+1)^2}dx \ .$$

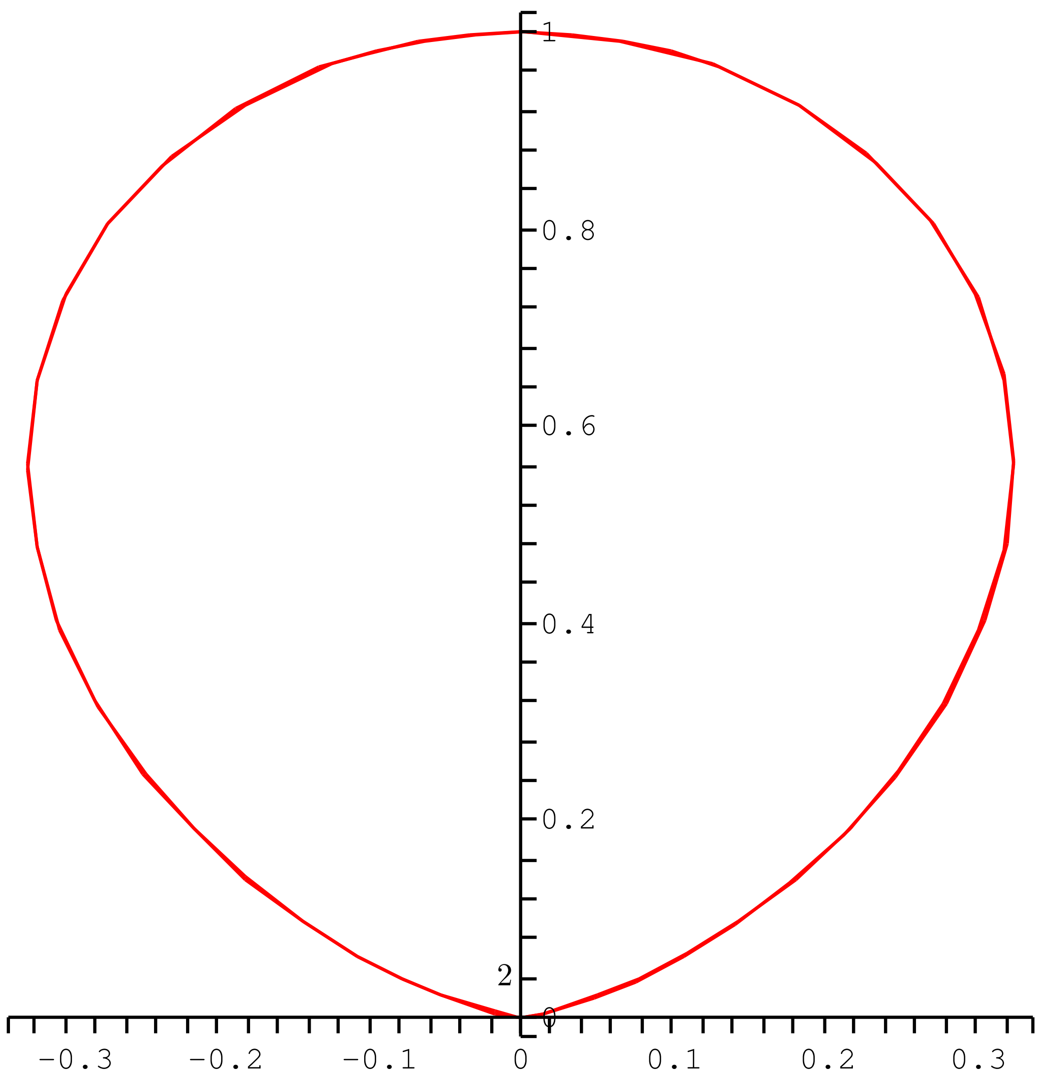
Substituting $z = (x+1)/\sqrt{2}$ yields

$$4\sqrt{2}\int_{1/\sqrt{2}}^{1} \sqrt{1-z^2}dz$$

and finally substituting $z = \sin(\theta)$ results in the integral

$$4\sqrt{2}\int_{\pi/4}^{\pi} \cos^2(\theta)d\theta \ = \frac{3\pi}{\sqrt{2}} - \sqrt{2} \ .$$

Problem 3:



The area is given by the integral

$$\int_0^{\pi} \left(\int_0^{\sin^3(\theta)} r dr \right) d\theta = \frac{1}{2} \int_0^{\pi} \sin^6(\theta) d\theta = \frac{15\pi}{96} \ .$$