Pratice Test 1A for Math2605, Spring 2004

Problem 1

Consider the function

$$f(x, y) = x^4 + y^4 - 4xy \ .$$

a) Calculate the gradient at the point $\mathbf{x_0} = (2^{1/4}, 0)$.

b) Find the equation for the line tangent to the curve $f(x, y) = f(\mathbf{x_0})$.

c) Find all the points on the curve $f(x, y) = f(\mathbf{x_0})$ where the tangent line is parallel to the x-axis.

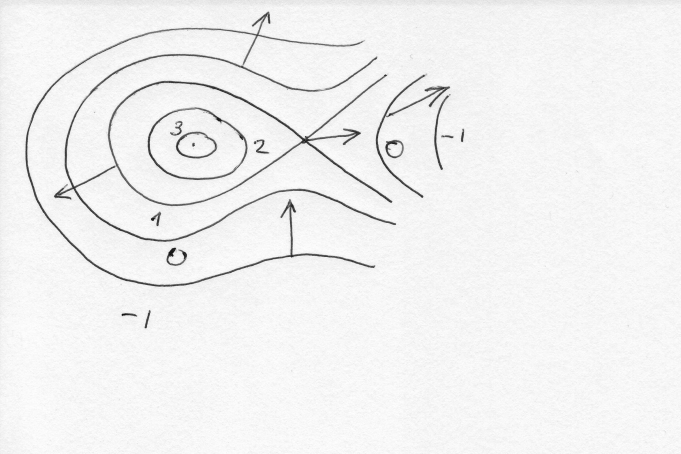
d) Find all the points on the curve $f(x, y) = f(\mathbf{x_0})$ where the tangent is parallel to the y axis.

e) Find points on the curve where the tangent line is parallel to the line $y = x$ and also those where the tangent line is parallel to the line $y = -x$. (Look at the symmetry of the curve!)

f) Sketch the curve, using the information about the tangents. Do not forget to take the symmetry of the curve into consideration.

Problem 2

a) Below is a plot of level sets of a function. Which one of the arrows could be the gradient of that function at that point?



b) Calculate the critical points of the function

$$f(x, y) = \frac{xy}{(1 + x^2 + y^2)^2}.$$

c) Calculate the Hessian at two of these critical points and decide their type, and whether they are stable or unstable.