

Practice Test 1A for Math2605, Fall 2004

Consider the function

$$f(x, y) = \frac{x^2 - y^2}{(1 + x^2 + y^2)^2} .$$

Problem 1

- Find the equation for the plane tangent to the graph of the function at the point $(-1, 1)$.
- Find the line (in parametrized form) that is tangent to the level curve at the point $(1, 0)$.
- Consider the level curve $f(x, y) = \frac{1}{8}$. Find the points where the tangent line is horizontal.

Problem 2

- Find all the critical points of this function.
- Find the Hessian of this function at the critical points.
- What is the type of each critical point?
- Draw a few level curves of this function in the vicinity of the critical points.

Problem 3

Consider the map

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} x^3 - 3xy^2 \\ -y^3 + 3yx^2 \end{bmatrix} .$$

- Use the initial value $\mathbf{x}_0 = (0, -1)$ and use one step in Newton's method towards solving the equations

$$\mathbf{F}(\mathbf{x}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} .$$

- Check if the new point \mathbf{x}_1 yields an improvement.

Extra credit:

The curve C given by intersecting the sphere $x^2 + y^2 + z^2 = 3$ with the ellipsoid $x^2 + 2y^2 + 3z^2 = 6$ passes through the point $(1, 1, 1)$. Find the line (in parametrized form) that is tangent to C and that passes through that point.