Pratice Test 2A for Math2605, Fall 2004

Problem 1

Find the maximum value of the function

$$\frac{1}{1+x^2+(y-1)^2}$$

on the set given by all pairs (x, y), such that $x^2 - y^2 \geq 1$. Find all the points where the maximal value is attained.

Problem 2

a) Apply one step of the Jacobi iteration for diagonalizing the matrix

$$\begin{bmatrix} 2 & 0.1 & 1 \\ 0.1 & 4 & 0.2 \\ 1 & 0..2 & 2 \end{bmatrix}$$

Make sure that you do it in such a fashion that the new matrix is almost diagonal.

b) Give upper and lower bounds on the eigenvalues as accurately as you possibly can.

Problem 3

Consider the matrix

$$A = \begin{bmatrix} 2 & -6 \\ 2 & 9 \\ -8 & -6 \end{bmatrix} .$$

a) Find the singular value decomposition for this matrix.

b) Find the generalized inverse A^+ for this matrix

c) Find the least square least length solution of the equation $A\mathbf{x} = \mathbf{b}$ where

$$\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} .$$

d) Find $A_{(1)}$ the best rank one approximation of A.

Problem 4

a) Diagonalize the matrix

$$\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} .$$

b) Find the Schur factorization of this matrix.

Problem 5: True or False : a) Every matrix A, with distinct eigenvalues can be written in the form $A = QDQ^T$ where Q is a rotation and D is diagonal.

b) It takes $n-1$ steps steps to reach a Schur factorization for an $n \times n$ matrix.

c) Every matrix can be written in the form $A = VDU^T$ where V and U are rotations.

d) Every matrix with distinct eigenvalues can be diagonalized.

e) There is an iterative method for diagonalizing symmetric matrices and it takes finitely many iterations to arrive at its diagonal form.