Preptest 2B for Calculus III for CS-Majors, Math 2605A1-2
March 11, 2004

Name:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 **minutes.** Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: Find the maximum and the minimum of the function $f(x,y) = x^3 - 3xy^2 + 4(x^2 - y^2)$ on the disk $x^2 + y^2 \leq 1$. Find all the points where the maximum and minimum are attained.

II: Consider the matrix

$$A = \begin{bmatrix} 6 & 0 & 1 & 8 \\ 0 & 3 & 0 & 7 \\ 1 & 0 & 1 & 2 \\ 8 & 7 & 2 & -6 \end{bmatrix}.$$

a) Calculate $\mathrm{Off}(A)$.

b) Run one step of the Jacobi iteration for diagonalizing the matrix A. Pick the 2×2 submatrix in such a fashion that $\mathrm{Off}(GAG^t)$ is as small as possible. What is the value of $\mathrm{Off}(GAG^t)$ after the first Jacobi itereation? Calculate the Givens rotation G and the matrix GAG^t.

III: Let $A = \begin{bmatrix} 2 & -6 \\ 2 & 9 \\ -8 & -6 \end{bmatrix}$.

a) Find a singular value decomposition $A = VDU^t$ of A.

b) Find the generalized inverse A^+ of this matrix.

c) Find the least square-least length solution of the equation $A\mathbf{x} = \mathbf{b}$ where

$$\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

d) Find $A_{(1)}$, the best rank one approximation of A.

IV: Consider the matrix

$$B = \begin{bmatrix} 1 & 1+2t & t \\ 1+2t & 1 & 5t \\ t & 5t & 2 \end{bmatrix}.$$

a) Calculate the first order Taylor polynomials of the eigenvalues $\mu_i(t)$ of the matrix B.

b) Give an estimate for the difference of the first order Taylor polynomial and the actual eigenvalues when $t = 0.1$.

V: Using the point $(1, 1)$ as the initial guess, calculate one step in Newton's method for solving the system of equations

$$f(x, y) = x^2 + xy + y^2 - 4 = 0 \qquad g(x, y) = x^3 - 2xy^2 + 2 = 0 \ . \tag{1}$$

Check the accuracy of the new point $\mathbf{x_1}$ by substituting it in the above equations (1).