

## Practice Test 3 for Math2605, Fall 2007

### Problem 1

For the matrix

$$A = \begin{bmatrix} 4 & 8 \\ 3 & 2 \end{bmatrix}$$

- Find the  $QR$  factorization of  $A$ .
- The Schur factorization.
- Compute  $e^{At}$ .

**Problem 2** Consider the rotation matrix

$$Q = \frac{1}{45} \begin{bmatrix} 40 & -5 & 20 \\ 13 & 40 & -16 \\ -16 & 20 & 37 \end{bmatrix}$$

- Find the axis of rotation  $\mathbf{u}$  and the angle of rotation  $\theta$ .
- Find  $B$  so that  $Q = e^{\theta B}$ .
- Find a rotation  $R$  so that  $Q = R^2$ .
- Find the family of rotations that interpolate the identity and  $Q$ .

**Problem 3** Consider the differential equation  $x'' = x' - x + x^3$ .

- Write this equation as a first order system.
- Find all the critical points.
- Find the type of the critical points and decide whether they are linearly stable or unstable.
- Decide which ones are stable or unstable for the nonlinear system and determine their possible types.

**Problem 4** Consider the curve

$$x(t) = \frac{1}{2}t^2 - \frac{1}{3}t^3, \quad y(t) = \frac{1}{2}t^2 + \frac{1}{3}t^3, \quad 1 \leq t \leq 2.$$

- Find the velocity for all values of  $t$ .
- Find the unit tangent vector  $\mathbf{T}(t)$ .
- Find the length of the curve.
- Rewrite the curve in terms of the length parametrization  $s$ .
- Find the curvature  $\kappa(s)$ .