Solutions for Quiz 1 for Calculus ++, Math 2605 T1-2, September 1, 2011

Name:

This quiz is to be taken without calculators and notes of any sorts. The allowed time is 20 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: Given the two planes $2x + y + 2z = 1$ and $2x - 2y - z = -2$.
a) (3 points) Find the intersection of the two plane in parametrized form.
We have to solve the system of equations

$$2x + y + 2z = 1\ ,\ 2x - 2y - z = -2\ .$$

The augmented matrix is

$$\begin{bmatrix} 2 & 1 & 2 & | & 1 \\ 2 & -2 & -1 & | & -2 \end{bmatrix}$$

and row reduction leads to

$$\begin{bmatrix} 2 & 1 & 2 & | & 1 \\ 0 & 1 & 1 & | & 1 \end{bmatrix}\ .$$

Back substitution yields $z = t, y = 1 - t, x = -t/2$. Thus line of intersection is given by

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \frac{t}{2}\begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}\ .$$

b) (2 points) Find the angle between the two planes, i.e., the angle between the normal vectors of the two planes.
Calculating the dot product

$$\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = 4 - 2 - 2 = 0.$$

Thus the planes are perpendicular to each other.
II: Consider the line $t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and the vector $\vec{a} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ in the 2-dimensional plane.
a) (2 points) Draw a picture of the line and the vector and indicate in this picture the component of $\vec{a}$ that is parallel to the line and the one perpendicular to the line.

b) Compute the length of both components (2 points). What is the distance between the tip of the vector $\vec{a}$ and the line (1 point).

Set $\vec{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The the component of $\vec{a}$ parallel to $\vec{u}$ is

$$\frac{\vec{a} \cdot \vec{u}}{|\vec{u}|^2} \vec{u} = \frac{3}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

whichm has length $3/\sqrt{2}$.

The component perpendicular to $\vec{u}$ is

$$\vec{a} - \frac{\vec{a} \cdot \vec{u}}{|\vec{u}|^2} \vec{u} = \frac{1}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

which has length $1/\sqrt{2}$. Hence the distance is $1/\sqrt{2}$.