

## Quiz 2 for Calculus ++, Math 2605 T1-2, September 15, 2011

**Name:**

This quiz is to be taken without calculators and notes of any sorts. The allowed time is 20 minutes. Provide exact answers; not decimal approximations! For example, if you mean  $\sqrt{2}$  do not write 1.414...

**I:** a) (2 points) Calculate the gradient of the function  $f(x, y) = y - x^4/4$ .

$$\begin{bmatrix} -x^3 \\ 1 \end{bmatrix}$$

b) (2 points) At the point  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ , find the direction of largest increase of the function  $f(x, y)$ . (Just give the direction as a unit vector).

$$\frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

**II:** (2 points) Does the function  $g(x, y) = e^{x^2}y$  have critical points, i.e., the points where the gradient vanishes?

$$\nabla g = e^{x^2} \begin{bmatrix} 2xy \\ 1 \end{bmatrix}$$

There are no critical points.

**III:** (4 points) a) Find the points on the curve  $y^2 - x^4 = 1$  where the tangent line is horizontal.

The gradient of the function  $y^2 - x^4$  is

$$\begin{bmatrix} -4x^3 \\ 2y \end{bmatrix}$$

The gradient must point in the  $y$  direction and hence  $-4x^3 = 0$  or  $x = 0$  But this means that  $y^2 = 1$ . Hence the points are

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$