Quiz 4 for Calculus ++, Math 2605 T1-2, November 10, 2011

Name:

This quiz is to be taken without calculators and notes of any sorts. The allowed time is 20 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$.

I: a) (3 points) Find the Householder reflection that maps the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ to a multiple of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

The multiple is 5 and hence $\mathbf{u}$ is proportional to

$$\begin{bmatrix} 5 - 3 \\ 0 - 4 \end{bmatrix}$$

so that

$$\mathbf{u} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -2 \end{bmatrix} .$$

$$M = I - 2\mathbf{u}\mathbf{u}^T = \frac{1}{5} \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix} .$$

b) (2 points) Find the QR factorization of the matrix $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$.

$$Q = MA = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix} , R = M .$$

II: (3 points) The matrix A in Problem I b) has the eigenvalues $5, -1$ with the corresponding eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$. Use this to solve the initial value problem $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

$$\mathbf{x}(t) = ae^{5t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + be^{-1} \begin{bmatrix} 1 \\ -2 \end{bmatrix} ,$$

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

from which $a = b = 1$ follows.

III: (2 points) Write the second order differential equation $x'' = 2x' - x^2$ as a first order system.

Set $y = x'$ so that with $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\mathbf{x}' = \begin{bmatrix} y \\ 2y - x^2 \end{bmatrix} .$$