Final Exam for Calculus II, Math 1502, December 15, 2010

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given. **Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.**

Problem	Score
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
XI	
XII	
Total	

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Problems related to Block 1:

I: (15 points) Compute with an error less than 10^{-3}

$$\int_0^1 e^{x^4} dx \ .$$

II: a) (7 points) Compute the limit

$$\lim_{x \to 0} \frac{\log(1+x) + (1-x) - \cos x}{x^3}$$

b) (8 points) Does the improper integral

$$\int_0^1 \frac{1}{x^2} e^{-\frac{1}{x}} dx$$

exist? If yes, compute it.

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Problems related to Block 2:

III: a) (7 points) Is the series

$$\sum_{k=2}^{\infty} \frac{k^{\log k}}{(\log k)^k}$$

convergent?

b) (8 points) Find the interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} x^k$$

IV: (15 points) Solve the initial value problem

$$y' + (x+1)y = e^{-\frac{x^2}{2} - x} \ , \ y(0) = 1 \ .$$

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Problems related to Block 3:

V: (20 points) Find a one-one parametrization for the solution of the systems below provided the solution exists:

$$x + 2y - 6z = 2$$
$$2x + y + 3z = 1$$
$$3x + y + 7z = 1 \ .$$

$$x + 2y + 7z = 1$$
$$x + 3y + 12z = 1$$
$$-x + 4y + 23z = 1 \ .$$

VI: (15 points) A plane in $\mathbb{R}^3$ has the parametric representation $\vec{x}(s,t) = \vec{x}_0 + s\vec{v}_1 + t\vec{v}_2$ where $\vec{x}_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. Find an equation for this plane.

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Problems related to Block 4:

VII: (20 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 3 & 4 & 6 \end{bmatrix}$$

a) Find a basis for $Ker(A)$ and a basis for $Img(A)$,

b) Find a basis for $Ker(A^T)$ and $Img(A^T)$.

VIII: (15 points) Find the QR factorization of the matrix

$$\begin{bmatrix} 1 & 6 & 4 \\ 2 & 3 & -1 \\ 2 & -6 & -10 \end{bmatrix}$$

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Problems related to Block 5:

IX: (15 points) Consider the curve given by the equation

$$8x^2 + 6xy = 1 \ .$$

a) What is the type of the curve? Is it an ellipse or hyperbola?

b) Graph the curve in a qualitative fashion below. Indicate in the figure the directions of the eigenvectors.

X: (15 points) Find the eigenvalues and eigenvectors of the following matrices. What is their algebraic multiplicity, what is their geometric multiplicity?

$a)$ $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$b)$ $\begin{bmatrix} 2 & 9 \\ 4 & 7 \end{bmatrix}$

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XI: (20 points) Using the superposition principle, solve the initial value problem given by the system

$$\begin{aligned} x' &= 2x + 9y \\ y' &= 4x + 7y \\ x(0) = 1 \ , \ y(0) &= 2 \end{aligned} \tag{0.1}$$

XII: (20 points) A sequence of numbers $a_n, n = 0, 1, 2, \ldots$ satisfies the recursion relation

$$a_{n+1} = 5a_n - 4a_{n-1} \ , n = 0, 1, 2, \ldots$$

with the initial condition $a_0 = 1, a_1 = 2$. Find an expression for a_n for arbitrary n.