Practice Final Exam for Calculus II, Math 1502, December 7, 2012

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given. **Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.**

Problem	Score
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
Total	

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I: (15 points) Compute with an error less than 10^{-3}

$$\int_0^1 3^{x^2}\,dx \ .$$

II: a) (7 points) Compute the limit

$$\lim_{x \to \pi/2} \left[\tan(x) + \frac{1}{x - \pi/2} \right]$$

b) (8 points) Does the improper integral

$$\int_0^\infty \frac{1}{\sqrt{x} + e^x} dx$$

exist?

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III: a) (7 points) Is the series

$$\sum_{k=3}^{\infty} \frac{1}{k \log(k) \log(\log(k))}$$

convergent?

b) (8 points) Find the interval of convergence of the power series

$$\sum_{k=1}^{\infty} k^{\log k} x^k$$

IV: (15 points) Solve the initial value problem

$$y'' - 4y' + 5y = 0 \ , \ \ y(0) = 1 \ , y'(0) = 2 \ .$$

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V: (20 points) Consider the three vectors

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \; \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \; \vec{v}_3 = \begin{bmatrix} 1 \\ -2 \\ a \end{bmatrix}.$$

Determine all the values for a and b for which the vector

$$\vec{b} \begin{bmatrix} b \\ 2 \\ -1 \end{bmatrix}$$

is in the span of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

Determine all the values for a and b for which the vector $\vec{b}$ is a unique linear combination of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

VI: ((20 points) Find the least square solution for the system $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -1 & -4 \\ 1 & 3 & 0 \\ 1 & 3 & 6 \\ 1 & -1 & 2 \end{bmatrix}$$

and

$$\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Solve the problem in two ways, once using the normal equations and then using the QR factorization.

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VII: (15 points) Consider the matrix

$$A = \begin{bmatrix} 2 & 3 & 5 & 6 \\ 1 & 0 & 1 & 3 \\ 4 & 1 & 5 & 12 \\ 2 & 1 & 4 & 7 \end{bmatrix}$$

Find a basis for the column space of A, for the Null space of A as well as for the column space of A^T and the Null space of A^T. Try do this with as little computation as possible.

VIII: (15 points) Solve the recursive relation, i.e., find a_n for arbitrary values of n,

$$a_{n+1} = 4a_n - 3a_{n-1}$$

with $a_0 = a_1 = 1$.

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IX: (15 points) Graph the curve given by the equation

$$66x^2 - 24xy + 59y^2 = 25\ .$$

X: (15 points) Show that

$$\frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

is an eigenvector for the matrix

$$A = \frac{1}{9}\begin{bmatrix} 17 & -2 & -2 \\ -2 & 14 & -4 \\ -2 & -4 & 14 \end{bmatrix}\ .$$

Find all the eigenvalues and eigenvectors. Find an orthonormal basis so that the matrix A is diagonal.

Show that

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

is an eigenvector of the matrix

$$\frac{1}{2}\begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 1 & 1 & 4 \end{bmatrix}$$

Diagonalize this matrix.