Practice Test 4D for Calculus II, Math 1502, November 7, 2012

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.
Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

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I: (20 points) a) Compute a determinant for deciding whether the vectors

$$\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -7 \end{bmatrix}$$

are linearly independent or not.

b) Consider the matrix

$$A = \frac{1}{3} \begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

Compute $\det A^4$.

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II: (20 points) Consider the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 & 5 \\ 3 & 1 & 3 & 4 \\ -2 & 4 & -16 & 16 \end{bmatrix}$$

a) Find a basis for the Column Space.

b) Without doing any further computations, what is the dimension of the Column Space of A and what is the dimension of the Null space of A?

c) Find a basis for the Null Space.

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III: (20 points) a) Is the vector

$$\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

an eigenvector of the matrix

$$\begin{bmatrix} 3 & 6 & 7 \\ 3 & 2 & 7 \\ 5 & 6 & 4 \end{bmatrix} \ ?$$

If yes, what is the corresponding eigenvalue?

b) For the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

find an invertible matrix V and a diagonal matrix D such that $A = VDV^{-1}$ or explain why it cannot be done.

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IV: (20 points) Consider the difference equation

$$a_{n+1} = 2a_n + 3a_{n-1} \ , \ a_0 = 1, a_1 = 1 \ .$$

Find and expression for a_n for all $n = 1, 2, \ldots$.

V: (20 points) Which of the following matrices is diagonalizable? If it is , diagonalize it.

$$a) \quad \begin{bmatrix} -10 & 9 \\ -16 & 14 \end{bmatrix}$$

What is the algebraic multiplicity and the geometric multiplicity of each eigenvalue?

$$b) \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

What is the algebraic multiplicity and the geometric multiplicity of each eigenvalue?