1 Topics for the Final Exam

1.1 Calculus

Taylor's theorem including remainder. You should be able to apply it to compute certain quantities with a given accuracy.

l'Hopital's rule, Improper integrals, Convergence of series using the root-, ratio, comparison and limiting comparison test. The integral test) Alternating series including the error error bound for partial sums. You should know a few example, like the geometric sums and series as well as how to sum telescoping series.

Taylor series and power series, absolute convergence, conditional convergence, radius of convergence, interval of convergence, differentiation and integration of power series.d second

First order and second order linear differential equations, and applications to tank problems. Separation of variables.

1.2 Linear Algebra

Vectors and scalars, linear dependence and independence, spanning sets. Row reduction, pivots free variables, inverses. You should know how to use row reduction to answer questions about linear dependence and linear independence as well whether a vector is in the span of a given set of vectors.

Linear transformations, matrix associated with a linear transformation, composition of linear transformations and matrix products. Linear transformations that are onto and linear transformations that are one-to-one.

Determinants of 2×2 and 3×3 matrices. How to compute determinants with row reduction.

$$\det A^T = \det A \ , \ \det(AB) = \det A \det B$$

Subspaces, basis and dimension. Column space of a matrix, Null space of a matrix,
# of columns of a matrix = dimension of Null space + dimension of Column space.
You should be able to find a basis for the column space as well a basis for the Null space.

Definition of eigenvectors, Diagonalization of matrices,

$$A = VDV^{-1}$$

If A is $n \times n$ then one needs n linearly indpendent eigenvectors. If eigenvalues are distinct then matrix can be diagonalized. algebraig multiplicity, geometric multiplicity, if the the two numbers are the same for all eigenvalues then the matrix is diagonalizable. Application to solving difference equations

$$a_{n+1} = aa_n + ba_{n-1} \ , a_0 = \alpha \ , a_1 = \beta.$$

Dot product, length of vectors and angle between vectors, Schwarz's inequality, triangle inequality. Orthogonal complements, ortho-normal basis for subspaces, Gram- Schmidt procedure, Projections, best approximation of a vector by a vector belonging to a subspace. Least square approximations QR factorization of a matrix. The orthogonal complement of the column space of a matrix A is the Null space of the matrix A^T. Orthogonal matrices.

Symmetric matrices, are always diagonalizable, eigenvector can be chosen to be an orthonormal basis.

$$A = UDU^{-1} = UDU^T$$

Quadratic forms, sketch curves of the form $ax^2 + 2bxy = cy^2 = 1$ and decided whether the curve is an ellipse or a hyperbola.