Test 3 for Calculus II, Math 1502 H1-H5 , October 23, 2012

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

Name:

Section:

Name of TA:

I: Consider the system of equations

$$\begin{aligned} x + 2y + uz &= 1 \\ -x + z &= v \\ 5x + 6y + 7z &= 1 \end{aligned}$$

For which values of u and v does this system have a) (7 points) no solution, b) (7 points) exactly one solution, c) (6 points) infinitely many solutions?

Augmented matrix

$$\begin{bmatrix} 1 & 2 & u & | & 1 \\ -1 & 0 & 1 & | & v \\ 5 & 6 & 7 & | & 1 \end{bmatrix}$$

Row reduction

$$\begin{bmatrix} 1 & 2 & u & | & 1 \\ 0 & 2 & 1+u & | & 1+v \\ 0 & -4 & 7-5u & | & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & u & | & 1 \\ 0 & 2 & 1+u & | & 1+v \\ 0 & 0 & 9-3u & | & -2+2v \end{bmatrix}$$

$u = 3, v \neq 1$: No solution
$u = 3, v = 1$: infinitely many solutions.
$u \neq 3$ and v arbitrary: exactly one solution.

Name:

Section:

Name of TA:

II: a) (10 points) Find the product BA where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 2 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

By the way:

$$AB = \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & 2 \\ -2 & 2 & 8 \end{bmatrix}$$

b) (10 points) Compute $A^3 = AAA$ where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} .$$

$$A^3 = A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} .$$

Name:

Section:

Name of TA:

III: (20 points) Let $T : \mathcal{R}^3 \to \mathcal{R}^2$ be a linear transformation with the property that

$$T(\vec{e}_1 + \vec{e}_2 + \vec{e}_3) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \ , \ T(\vec{e}_2 + \vec{e}_3) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ , \ T(\vec{e}_3) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \ .$$

Find the matrix A associated with T, i.e. the matrix A that satisfies $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathcal{R}^3$.

$$T(\vec{e}_2) = T(\vec{e}_2 + \vec{e}_3) - T(\vec{e}_3) = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$T(\vec{e}_1) = T(\vec{e}_1 + \vec{e}_2 + \vec{e}_3) - T(\vec{e}_2 + \vec{e}_3) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Hence

$$A = [T(\vec{e}_1) \ , \ T(\vec{e}_2) \ , \ T(\vec{e}_3)] = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \ .$$

Name:

Section:

Name of TA:

IV: (10 points) Are the following three vectors in $\mathcal{R}^4$ linearly independent?

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

Row reduction:

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

The vectors are linearly independent (pivot in every column).
b) (10 points) Is the vector

$$\vec{b} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

in the span of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$?

Augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \end{array}\right]$$

Row reduction:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{array}\right], \left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

The vector $\vec{b}$ is not in the span of $\vec{v}_1, \vec{v}_2 \vec{v}_3$.

Name:

Section:

Name of TA:

V: True or false (each problem is worth 5 points, no partial credit):

a) If any pair out of three vectors is linearly independent, then the three vectors are linearly independent.
FALSE
b) If A is a matrix and the system of equations $A\vec{x} = \vec{b}$ has a solution for all vectors $\vec{b}$ then every row has a pivot.
TRUE
c) If $\vec{v}_1, \ldots, \vec{v}_p$ are vectors in $\mathcal{R}^n$, and $p > n$, then these vectors are linearly dependent.
TRUE
d) A system of linear equations can have exactly two solutions.
FALSE