Test 4 for Calculus II, Math 1502 H1-H5 , November 13, 2012

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This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

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I: (15 points) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & -1 & 2 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & -5 & 4 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 0 & 0 & 1 \\ 0 & -5 & 0 & 2 & 1 & -4 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{4}{5} & \frac{2}{5} & -\frac{3}{5} \\ 0 & 1 & 0 & -\frac{2}{5} & -\frac{1}{5} & \frac{4}{5} \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

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II: a) (10 points) Find a basis for the Null Space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 3 & 5 & 14 \\ -1 & 4 & 14 & 4 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & -1 & -3 & -2 \\ 0 & 6 & 18 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & -1 & -3 & -2 \\ 0 & 1 & 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Any vector in the Null Space of A is given by

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -4 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

The vectors

$$\begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

are a basis for the Null Space of A.

b) (5 points) What is the dimension of the Null Space of the matrix A?

2 dimensional

c) (5 points) What is the rank of the matrix A?

The rank is 2.

d) (10 points) Find a basis for the column space for the matrix A.

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

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III: a) (15 points) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Eigenvalues, Eigenvectors:

$$5 \ , \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$-1 \ , \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

b) (5 points) A 2×2 matrix B satisfies the equation

$$B^2 - B - 6I_2 = 0 \ ,$$

where I_2 is the 2×2 identity matrix. Find the eigenvalues of the B.

Any eigenvector $\vec{v}$ of B satisfies the equation $B\vec{v} = \lambda\vec{v}$ and hence

$$B^2\vec{v} - B\vec{v} - 6\vec{v} = (\lambda^2 - \lambda - 6)\vec{v} = \vec{0}$$

Hence,

$$\lambda^2 - \lambda - 6 = 0$$

and the eigenvalues are $3, -2$.

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IV: (15 points) Using eigenvalues and eigenvectors, compute a closed expression for A^k for any integer k where

$$A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} .$$

Eigenvalues/Eigenvectors:

$$0 \ , \begin{bmatrix} 1 \\ 1 \end{bmatrix} \ , \ 4 \ , \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$A^k = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 4^k \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4^k & -4^k \\ -4^k & 4^k \end{bmatrix} .$$

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V: The matrix

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

has the eigenvalue 4.

a) (5 points) Find the corresponding eigenvector.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

b) (10 points) Find the other eigenvalues (You do not have to compute any eigenvectors).
Characteristic polynomial:

$$-\lambda^3 + 4\lambda^2 + 7\lambda - 28 = (\lambda - 4)(-\lambda^2 + 7)$$

Thus, the other eigenvalues are $\sqrt{7}$ and $-\sqrt{7}$.
c) (5 points) Is this matrix diagonalizable? (You do not have to make any further computations!)
Yes, the eigenvalues are distinct and hence there are three linearly independent eigenvectors.