Homework 1 , due Tuesday September 10

I: a) Show that the vectors $e_k = (0, 0, \dots, 1, 0, 0 \dots)$ where the number 1 is in the k-th position, form a Schauder basis in ℓ^p for $1 \le p < \infty$.

b) Show that the vectors e_k above are not a Schauder basis for ℓ^∞.

II: Show that the subspace $c_0 \subset \ell^\infty$ consisting of sequences of scalars that converge to zero, is a Banach space in the norm inherited from ℓ^∞.

III: Let $T : C[0,1] \to C[0,1]$ be defined by

$$Tx(t) = \int_0^t x(\tau) d\tau \ .$$

Find $R(T)$ and $T^{-1} : R(T) \to C[0,1]$. Is it bounded?

IV: Consider the operator $T : \ell^\infty \to \ell^\infty$ defined by $Tx = (\eta_i), \eta_i = \frac{\zeta_i}{i}, (\zeta_i) = x$.

a) Show that it is linear and bounded linear operator.

b) Is the range $R(T)$ closed?

c) Find the inverse operator $T^{-1} : R(T) \to \ell^\infty$. Is it bounded?