Homework 2 , due Thursday October 3

I: Let A and $B \supset A$ be non-empty subsets of an inner product space X. Show that

$$a)\ \ A \subset A^{\perp\perp},\ \ b)\ \ B^{\perp} \subset A^{\perp},\ \ c)\ \ A^{\perp} \subset A^{\perp\perp\perp}\ .$$

II: Show that that a subspace Y of a Hilbert space H is closed in H if and only if $Y = Y^{\perp\perp}$.

III: If $M \neq \emptyset$ is any subset of a Hilbert space H, show that $M^{\perp\perp}$ is the smallest closed subspace of H that contains M, i.e., if $Y \subset H$ is any closed closed subspace that contains M then $M^{\perp\perp} \subset Y$.

IV: If z is any fixed element in an inner product space X, show that $f(x) = \langle x, z\rangle$ defines a bounded linear functional on X with norm $\|z\|$. If the mapping $X \to X'$ given by $z \to f$ is surjective, show that X must be a HIlbert space.