Homework 4 , due Thursday November 14

I: Let $\mathcal{H}$ be a Hilbert space and let $T : \mathcal{H} \to \mathcal{H}$ be a linear operator such that for all $x, y \in \mathcal{H}$

$$\langle Tx, y \rangle = \langle x, Ty \rangle \ .$$

Prove that T is bounded.

II: Let $\mathcal{H}$ be a complex Hilbert space and $A \subset \mathbb{C}$ a compact set. Construct a linear bounded operator T such that $\sigma(T) = A$.

III: Consider the finite dimensional space $\mathbb{C}^n$ endowed with the inner product

$$\langle z, w \rangle = \sum_{j=1}^{n} z_j \overline{w_j} \ .$$

On this space, let A be any self-adjoint operator. Find the spectral family for this operator.

IV: Solve problem 4 on page 385 in Kreyszig.

V: Solve problem 9 on page 475 in Kreyszig.