Final exam , due Tuesday December 10

Try to do these problems by yourself. If there is something you do not understand, you may discuss this with me or with your fellow students. However, do not copy the work of others. Print this page, sign it and staple it to your work. With your signature you affirm that you did not copy your work of others.

Signature:

I: (10 points) Please do problem 9 on page 291 in Kreyszig.

II: (10 points) Please do problem 4 on page 455 in Kreiszig.

III: (20 points) Let H be a Hilbert space and A, B bounded positive operators defined on H such that $A^{-1} : H \to H$ exists (and hence is bounded) and $A \leq B$. Show that a) $B^{-1} : H \to H$ exists and is bounded and that b)

$$A^{-1} \geq B^{-1}$$

IV: (20 points) On the space $C[0,1]$ with the norm $\|x\| = \max_{0 \leq t \leq 1} |x(t)|$, consider the linear operator

$$Tx(t) = \int_0^t k(t,s)x(s)ds$$

where $k(t,s)$ is a jointly continuous function on $[0,1] \times [0,1]$. Show that $\sigma(T) = \{0\}$.
(Hint: The function $k(t,s)$ satisfies $|k(t,s)| \leq M$ for some positive constant M (Why?). Use this to show that the Neumann series for the operator

$$(\lambda - T)^{-1} = \frac{1}{\lambda} \sum_{n=0}^{\infty} \left(\frac{T}{\lambda} \right)^n$$

converges in the operator norm for $\lambda \neq 0$.)