

PRINT Name:

PRINT Section:

PRINT Name of TA:

I: (25 points) Consider the function e^{-x} .

a) Find the 4-th order Taylor polynomial $P_4(x)$ for e^{-x} and the corresponding remainder in Lagrange form.

b) Using the above result compute an approximate value, call it A , for $\frac{1}{e}$

c) Give an estimate on how accurate the value computed in b) approximates $\frac{1}{e}$, i.e., give a bound on

$$\left| \frac{1}{e} - A \right| ,$$

using the remainder found in a).

PRINT Name:

PRINT Section:

PRINT Name of TA:

II: Decide whether the following series converge or diverge. State which convergence test you are going to use.

a) (8 points)

$$\sum_{k=0}^{\infty} \frac{[k!]^2}{(3k)!}$$

b) (8 points)

$$\sum_{k=1}^{\infty} \frac{3^{k^2}}{k!}$$

c) (9 points)

$$\sum_{k=1}^{\infty} (2 + (-1)^k) \left(1 - \frac{1}{k}\right)^{k^2}$$

PRINT Name:

PRINT Section:

PRINT Name of TA:

III: a) (9 points) Consider the alternating series

$$L = \sum_{k=0}^{\infty} (-1)^k 10^{-k^2}$$

Find the smallest value of N so that the N -th partial sum s_N satisfies $|L - s_N| < 10^{-16}$.

b) (8 points) Find the power series expansion for $\sinh x := \frac{1}{2}(e^x - e^{-x})$.

c) (8 points) Sum the series

$$\sum_{k=0}^{\infty} (k+2)2^{-k}$$

PRINT Name:

PRINT Section:

PRINT Name of TA:

IV: Find the interval of convergence of the following power series. State which convergence test you are going to use for computing the radius of convergence.

a) (7 points)

$$\sum_{k=0}^{\infty} \frac{\sqrt{k!}}{k^k} x^k$$

b) (8 points)

$$\sum_{k=1}^{\infty} (-1)^k \frac{1}{k^3} \left(\frac{x+3}{2} \right)^k$$

c) (10 points)

$$\sum_{k=1}^{\infty} \frac{3 + (-1)^k}{k} (x-1)^k$$

What function does this series represent in its open interval of convergence?