Test 3 for Calculus II, Math 1502 K1 - K6, October 23, 2013

PRINT Name:

PRINT Section:

PRINT Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given.

PRINT your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

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I: Consider the system of equations

$$\begin{aligned} x + 3y - 5z &= -1 \\ 2x + y + 5z &= 8 \\ x + 2y - 2z &= b \end{aligned}$$

a) (10 points) Using row reduction reduce this system to *echelon form.*

b) (2 points) For which values of b, if any, is the system consistent?

c) (2 points) For which values of b, if any, is there a unique solution?

d) (6 points) For which values of b, if any, are there infinitely many solutions? Compute all the solutions for these cases.

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II: Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}, \ \vec{v}_3 = \begin{bmatrix} 2 \\ -13 \\ -5 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

a) (14 points) Determine all the vectors $\vec{b}$ that can be written as a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

b) (6 points) Are the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ linearly independent?

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III: Consider the matrix

$$B = \begin{bmatrix} 1 & 2 & 1 & 5 & 1 \\ 3 & 6 & 1 & 11 & 7 \\ 1 & 2 & 2 & 7 & -1 \end{bmatrix}$$

a) (10 points) Using row operations, bring this matrix to *reduced echelon form.*

b) (4 points) Indicate in the matrix B the pivotal positions.

c) (6 points) The matrix B is the augmented matrix of the linear system

$$\begin{gathered} w + 2x + y + 5z = 1 \\ 3w + 6x + y + 11z = 7 \\ w + 2x + 2y + 7z = -1 \end{gathered}$$

find all the solutions of this system.

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IV: a) (10 points) Let $T : R^2 \to R^2$ be a linear transformation that maps the vector $\vec{e}_1$ to the vector $\vec{e}_1 + \vec{e}_2$ and the vector $\vec{e}_2$ to the vector $\vec{e}_1$. What is the matrix associated with T.

b) (10 points) The linear transformations $Q : R^2 \to R^2$ is obtained by first performing the shear transformation

$$S\vec{x} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \vec{x}$$

and then a rotation by $45°$. Find the matrix associated with Q.

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V: No partial credit: (5 points each) True or false:
a) A system of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$ vectors in R^n with $p > n$ are always linearly dependent.

b) If for a system of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$ in R^n every pair is linearly independent, then the whole system is linearly independent.

c) For a linearly dependent system of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p$ in R^n the vector $\vec{v}_1$ can always be expressed as a linear combination of the vectors $\vec{v}_2, \ldots, \vec{v}_p$.

d) For a given system of linear equations, the echelon form is unique.