Practice Test 4B for Calculus II, Math 1502, November 9, 2013

PRINT Name:

PRINT Section:

PRINT Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write 1.414 .... Show your work, otherwise credit cannot be given.

PRINT your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.

PRINT Name:

PRINT Section:

PRINT Name of TA:

I: a) (10 points) Let S_1 and S_2 be two subspaces of $\mathcal{R}^n$. Show that the intersection of these subspaces is again a subspace.

b) (10 points) Consider the subspacesS_1 given by the span of the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} , \vec{v}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

and S_2 which is given by the span of the vectors

$$\vec{w}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} , \vec{w}_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} .$$

Find the intersection of S_1 and S_2.

PRINT Name:

PRINT Section:

PRINT Name of TA:

II: a) (10 points) By computing the determinant of a matrix decide for which values of a the vectors

$$\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ a \\ 1 \end{bmatrix}, \begin{bmatrix} -10 \\ 6 \\ 6 \end{bmatrix}$$

are linearly dependent.

b) (10 points) By making as few computations as possible compute the determinant of the matrix

$$\begin{bmatrix} 5 & 6 & 6 & 6 & 6 \\ 3 & 4 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 & 0 \\ 4 & 5 & 5 & 5 & 4 \end{bmatrix}$$

PRINT Name:

PRINT Section:

PRINT Name of TA:

III: a) (10 points) Consider the matrices

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \text{ and } E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Compute the inverse of $E_1 \cdot E_2$.

b) (10 points) Given

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Find all the vectors $\vec{b} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ so that the equation

$$LU\vec{x} = \vec{b}$$

has a solution.

PRINT Name:

PRINT Section:

PRINT Name of TA:

IV: (20 points) Find the eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

(Hint: One eigenvector is easy to guess).

PRINT Name:

PRINT Section:

PRINT Name of TA:

V: (5 points) each. Prove or find a counterexample. Let A be an $n \times m$ matrix.

a) $Nul(A)$ is a subspace of $\mathcal{R}^n$.

b) $\dim Col(A)$ is less than the smaller of the numbers n and m.

c) If the dimension of $Col(A)$ is 1, then the dimension of $Nul(A)$ is $m - 1$.

d) If λ is an eigenvalue of A then $\dim Nul(A - \lambda I) = 1$.