Topics for Test 4

Subspaces of $\mathcal{R}^n$

You should know the definition of a subspace and the definition of a basis. The important theorem here is that two basis for a subspace $S \subset \mathcal{R}^n$ must have the same number of vectors. This number is the dimension of the subspace S. You should be able to decide whether a given set is a subspace and you should be able to decide whether a given set of vectors constitutes a basis for a subspace.

Column space and Null space of a matrix

You have to understand the what is the Column Space, $Col(A)$ of an $m \times n$ matrix A. This is a subspace of $\mathcal{R}^m$. Likewise the Null Space $Nul(A)$ of this matrix is a subspace of $\mathcal{R}^n$. You have to be able to compute bases for both of these subspaces using row reduction. The important theorem here is the that

$$\dim Col(A) + \dim Nul(A) = n$$

where n is the number of columns of the matrix A. You should understand how this fact is derived from row reduction. Remember that the $rank(A)$ is another word for the dimension of the column space of the matrix A.

Invertible matrices

This is a special subclass of $n \times n$ matrices. You have to understand the equivalence of the various statements, like ‘ the matrix is invertible’ if and only if the column vectors are linearly independent if and only if the columns span $\mathcal{R}^n$ etc. Please see page 112 in Lay. You should also be able to compute the inverse using row reduction. You have to know various formulas such as

$$(AB)^{-1} = B^{-1}A^{-1} \ .$$

Determinants

You have to understand the definition of determinants and how to compute them using co-factor expansion and row reduction. You also have to know the role played by determinants in computing volumes. An important relation is that for A, B $n \times n$ matrices

$$\det(A \cdot B) = \det(A)\det(B)$$

Eigenvalues eigenvectors

The definition of eigenvectors and eigenvalues is important. You should be able to calculate the eigenvalues for simple matrices and find the eigenvectors. It is important to understand the concept of eigenspace, i.e.,

$$Nul(A - \lambda I)$$

where λ is an eigenvalue. Naturally, this space is described using a basis.