Quiz 1, Math 1502 B1-B6, September 11, 2014

Name:

Section:

Name of TA:

Calculators are allowed. Carefully explain your procedures and answers. If there is not enough space, continue on the back of this page. **Write your name, your section number as well as the name of your TA on this page, this is very important.**

Problem 1 (10 points) a) (5 points) Estimate the error for the Trapezoid rule applied to

$$\int_0^2 x\sin(x)dx$$

with $N = 10$. Your bounds on the derivatives should be rigorous.

The first and second derivatives of the integrand are

$$\sin(x) + x\cos(x)\ ,\ 2\cos(x) - x\sin(x)\ .$$

Now for $x \in [0, 2]$,

$$|2\cos(x) - x\sin(x)| \le 2|\cos(x)| + x\sin(x) \le 4\ .$$

Therefore the error is bounded by

$$\frac{4}{12}\frac{2^3}{100} = \frac{8}{300}\ .$$

b) (5 points) Solve the ODE

$$y' + y/x = x^4\ ,\ y(1) = 7$$

An integrating factor is x and hence

$$(xy)' = xy' + y = x^5$$

which, upon integration, yields

$$y(x) = \frac{1}{6}x^5 + \frac{C}{x}\ .$$

With the initial condition

$$7 = y(1) = \frac{1}{6} + C$$

and so

$$y(x) = \frac{1}{6}x^5 + \frac{41}{6x}\ .$$

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Problem 2 (10 points) a) (4 points) Find the series for

$$\frac{x^2}{1-x^3}$$

$$x^2 \sum_{n=0}^{\infty} x^{3n} = \sum_{n=0}^{\infty} x^{3n+2} = x^2 + x^5 + x^8 + x^{11} + \cdots$$

which holds for $|x| < 1$. b) Do the following expressions converge? (you must explain why).

i) (3 points)

$$\int_1^{\infty} \frac{|\sin(x)|}{x^2} dx$$

The integral converges, because the direct comparison test yields

$$\frac{|\sin(x)|}{x^2} \le \frac{1}{x^2}$$

and

$$\int_1^{\infty} \frac{1}{x^2} dx$$

converges.

ii) (3 points)

$$\sum_{n=2}^{\infty} \frac{2^{n\ln(n)}}{n^n}$$

By the root test we have to compute

$$\lim_{n\to\infty} \frac{2^{\ln(n)}}{n} .$$

Because

$$\frac{2^{\ln(n)}}{n} = \frac{2^{\ln(n)}}{e^{\ln(n)}} = \left(\frac{2}{e}\right)^{\ln(n)}$$

we find

$$\lim_{n \to \infty} \frac{2^{\ln(n)}}{n} = 0$$

because

$$\frac{2}{e} < 1 \ .$$

Therefore the series converges.

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Problem 3 (10 points)
a) (5 points) Sum the series

$$\sum_{n=2}^{\infty} \frac{4}{n^2+2n}$$

Give an exact answer!

$$\frac{4}{n^2+2n} = \frac{4}{(n+2)n} = 2(\frac{1}{n} - \frac{1}{n+2})$$

Thus the sum is a telescoping sum and

$$2\sum_{n=2}^{N} \frac{2}{n^2+2n} = 2\sum_{n=2}^{N} \frac{1}{n} - \frac{1}{n+2} = 2\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{N+1} - \frac{1}{N+2}\right)$$

which in the limit as $N \to \infty$ yields $5/3$.

a) (5 points) Sum the series

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{2^k}{7^{k+1}}$$

Give an exact answer!

The sum can be written as

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{2^k}{7^{k+1}} = -\frac{1}{7}\sum_{k=1}^{\infty} (-1)^k \frac{2^k}{7^k} = -\frac{1}{7}\left(\sum_{k=0}^{\infty} (-1)^k \frac{2^k}{7^k} - 1\right)$$

which, using the formula for the geometric series ($2/7 < 1$!) yields

$$-\frac{1}{7}\left(\frac{1}{1+\frac{2}{7}} - 1\right) .$$

Although not necessary, this can be simplified to

$$\left(\frac{1}{7} - \frac{1}{9}\right) = \frac{2}{63} .$$

Another way of solving it (thanks Tom) is to write

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{2^k}{7^{k+1}} = \frac{2}{7^2}\sum_{k=1}^{\infty} (-1)^{k-1} \frac{2^{k-1}}{7^{k-1}} == \frac{2}{7^2}\sum_{k=0}^{\infty} (-1)^k \frac{2^k}{7^k}$$

$$= \frac{2}{7^2}\frac{1}{1+2/7} = \frac{2}{63}$$