Quiz 2, Math 1502 B1-B6, October 2, 2014

Name:

Section:

Name of TA:

Calculators are allowed. Carefully explain your procedures and answers. If there is not enough space, continue on the back of this page. **Write your name, your section number as well as the name of your TA on this page, this is very important.**

Problem 1 (10 points) Write

$$e^{-x^2} = \sum_{n=0}^{N} \frac{(-x^2)^n}{n!} + (-1)^{N+1} e^c \frac{x^{2N+2}}{(N+1)!}$$

where c depends on x and is between 0 and $-x^2$. Integrating this series yields

$$\int_0^2 e^{-x^2} dx = \sum_{n=0}^{N} (-)^n \frac{1}{n!} \int_0^2 x^{2n} dx + (-1)^{N+1} \int_0^2 e^c \frac{x^{2N+2}}{(N+1)!} dx$$

Hence

$$|\int_0^2 e^{-x^2} dx - \sum_{n=0}^{N} (-)^n \frac{2^{2n+1}}{n!(2n+1)}| < \int_0^2 e^c \frac{x^{2N+2}}{(N+1)!} dx$$

Now

$$e^c \le e^0 = 1$$

and hence

$$|\int_0^2 e^{-x^2} dx - \sum_{n=0}^{N} (-)^n \frac{2^{2n+1}}{n!(2n+1)}| < \int_0^2 \frac{x^{2N+2}}{(N+1)!} dx = \frac{2^{2N+3}}{(N+1)!(2N+3)}$$

For $N = 15$ we get that the remainder is less than

$$1.3 \cdot 10^{-5} \ .$$

So the sum

$$\sum_{n=0}^{15} (-)^n \frac{2^{2n+1}}{n!(2n+1)}$$

will do the job.

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Problem 2 (10 points) a) The ratio test applied to the series

$$\sum_{n=3}^{\infty} \frac{1}{3^n \ln(n)} (x-2)^n$$

yields

$$\frac{|x-2|}{3} < 1$$

for convergence. Hence the radius of convergence is 3. The series diverges at $x = 5$ and converges at $x = -1$ (alternating series).

b)

$$\lim_{x \to 0} \frac{e^{-x^2} - 1}{x^2} = \lim_{y \to 0} \frac{e^{-y} - 1}{y} = -1 \ .$$

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Problem 3 (10 points) The point we are looking for is the intersection of the line

$$(3, 5, 4) + t\langle 1, 2, 3\rangle$$

with the plane.

$$(3 + t) + 2(5 + 2t) + 3(4 + 3t) = 25 + 14t = -3$$

and so $t = -2$ and hence the point is $(1, 1, -2)$.

b)

$$\langle 1, 2, 3\rangle \times \langle 2, 0, 1\rangle = \langle 2, 5, -4\rangle \ ,$$

and

$$|\langle 2, 5, -4\rangle| = \sqrt{4 + 25 + 16} = \sqrt{45} = 3\sqrt{5} \ .$$

Extra credit: The point has to be on the line connecting the origin with the center of the sphere, i.e., of the form

$$t\langle 2, 3, 6\rangle = \langle 2t, 3t, 6t\rangle$$

for some t. To be on the sphere means

$$(2t - 2)^2 + (3t - 3)^2 + (6t - 6)^2 = 4$$

or

$$(t - 1)^2[4 + 9 + 36] = 4$$

which yields

$$t = 1 \pm \frac{2}{7} = \frac{9}{7}$$

We have to choose $t = 1 + \frac{2}{7}$, the other solution yields the closest point. Hence the point is

$$\frac{1}{7}(18, 27, 54)$$