Quiz 3, Math 1502 B1-B6, October 23, 2014

Name:

Section:

Name of TA:

Calculators are allowed. Carefully explain your procedures and answers. If there is not enough space, continue on the back of this page. **Write your name, your section number as well as the name of your TA on this page, this is very important.**

Problem 1 (10 points) a) (7 points) Find all the solutions of the system

$$\begin{gathered} x_1 + 2x_2 + 4x_3 = 8 \\ 2x_1 - 4x_3 = 0 \\ 3x_1 + x_2 - 3x_3 = 4 \end{gathered}$$

(Check your answer!)

Row reduction leads to

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

where x_3 is a free variable.

b) (3 points) The two vectors

$$\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

span a plane passing through the origin. Find a vector that is normal to this plane.

The cross product of the two given vectors yield a vector normal to the plane.

$$\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix}$$

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Problem 2: (10 points) For which values of a are the vectors

$$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} , \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} , \begin{bmatrix} 0 \\ a \\ 1 \end{bmatrix}$$

linearly independent?

Row reduce the system $A\vec{x} = \vec{0}$ where

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 0 & a \\ 1 & 1 & 1 \end{bmatrix} .$$

This leads to

$$(a+4)x_3 = 0 \ , \ x_2 = -3x_3 \ , \ x_1 = 2x_3$$

and hence, if $a \neq -4$ the system has only the trivial solution and the vectors are linearly independent. If $a = -4$ then x_3 is a free variable and

$$x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

is a non-trivial solution for all $x_3 \neq 0$ and the vectors are linearly dependent.

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Problem 3 (10 points) a) (5 points) Write the linear transformation

$$T(\vec{x}) = \begin{bmatrix} 2x_1 + 3x_2 \\ -x_1 + x_2 \\ 5x_1 \end{bmatrix}$$

in the form $T(\vec{x}) = A\vec{x}$ where A is a matrix.

The matrix is

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 5 & 0 \end{bmatrix}$$

and hence

$$T(\vec{x}) = \begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} .$$

b)(5 points) Given the linear transformation $T(\vec{x}) = \begin{bmatrix} x_1 - 2x_2 \\ -2x_1 + 4x_2 \end{bmatrix}$, is the vector $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ in the range of T?

We have to show that the system

$$x_1 - 2x_2 = 1 \ , \ -2x_1 + 4x_2 = -2$$

has a solution. The second equation is -2 times the first and hence the system has a solution.

Extra credit: (5 points) A linear transformation $T : R^2 \to R^2$ maps the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ to the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. What is the matrix associated with T, i.e., find a matrix A so that $T(\vec{x}) = A\vec{x}$.

We know that

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The vector $\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and since the transformation is linear we have that

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) - T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Hence

$$A = \left[T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right), T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right] = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}.$$