Practice Final Exam for Calculus II, Math 1502, December 2, 2013

Name:

Section:

Name of TA:

This test is to be taken without calculators and notes of any sorts. The allowed time is 2 hours and 50 minutes. Provide exact answers; not decimal approximations! For example, if you mean $\sqrt{2}$ do not write $1.414\ldots$. Show your work, otherwise credit cannot be given. **Write your name, your section number as well as the name of your TA on EVERY PAGE of this test. This is very important.**

Problem	Score
I	
II	
III	
IV	
V	
VI	
VII	
VIII	
IX	
X	
XI	
XII	
Total	

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I: (15 points) Compute with an error less than 10^{-3}

$$\int_2^3 e^{\frac{1}{x^2}}\,dx\ .$$

II: a) (7 points) Compute the limit

$$\lim_{x\to 0}\frac{e^x-\cos x-\sin x}{x^3}$$

b) (8 points) Does the improper integral

$$\int_0^1 \frac{1}{x^2}e^{\frac{1}{x}}\,dx$$

exist? If yes, compute it.

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III: a) (7 points) Is the series

$$\sum_{k=0}^{\infty}(-1)^k \frac{(k!)^2}{k^{2k}}$$

convergent? Is it absolutely convergent?

b) (8 points) Find the interval of convergence of the power series

$$\sum_{k=3}^{\infty}(-1)^k \frac{\ln k}{k}(x-2)^k$$

IV: (15 points) Solve the initial value problem

$$y' - \frac{1}{x^2}y = e^{-\frac{1}{x}}\ ,\ y(1) = \frac{2}{e}\ .$$

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V: (15 points) Consider the system of equations

$$2x + y + z = b$$
$$x + y - 2z = 2$$
$$x - y + az = -1$$

Determine all values for a and b for which this system has a) no solution, b) exactly one solution, c) infinitely many solutions. In the case b) and c) Compute all the solutions in terms of a and b.

VI: (15 points) Consider the three vectors

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \ \vec{v}_3 = \begin{bmatrix} 1 \\ -2 \\ a \end{bmatrix}.$$

Determine all the values for a and b for which the vector

$$\vec{b} \begin{bmatrix} b \\ 2 \\ -1 \end{bmatrix}$$

is in the span of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

Determine all the values for a and b for which the vector $\vec{b}$ is a unique linear combination of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

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VII: (15 points) Consider the matrix

$$\begin{bmatrix} 2 & 3 & 5 & 6 \\ 1 & 0 & 1 & 3 \\ 4 & 1 & 5 & 12 \\ 2 & 1 & 4 & 7 \end{bmatrix}$$

Find a basis for $Col(A)$ and for $Nul(A)$ as well as for $Col(A^T)$ and for $Nul(A^T)$. Try to do this with as little computation as possible.

VIII: (15 points) Diagonalize the matrices

$$a) \quad \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{bmatrix} .$$

$$b) \quad \begin{bmatrix} 6 & 9 \\ 4 & 11 \end{bmatrix}$$

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IX: (20 points) Solve the recursive relation, i.e., find a_n for arbitrary values of n,

$$a_{n+1} = 8a_n + 9a_{n-1}$$

with $a_0 = a_1 = 1$.

X: (20 points) Find the least square solution for the system $A\vec{x} = \vec{b}$ where

$$A = \begin{bmatrix} 1 & -1 & -4 \\ 1 & 3 & 0 \\ 1 & 3 & 6 \\ 1 & -1 & 2 \end{bmatrix}$$

and

$$\vec{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} .$$

Solve the problem in two ways, once using the normal equations and then using the QR factorization.

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XI: (15 points) Graph the curve given by the equation

$$11x^2 - 6xy + 19y^2 = 10 \ .$$

XII: (4 points each) Prove or find a counterexample:

a) Row operations do not change the column space of a matrix.

b) Row operations do not change the null space of a matrix

c) A 3×3 matrix has the eigenvalues $1, 2, 2$. Is it necessarily diagonalizable?

d) A real 3×3 matrix must always have at least one real eigenvalue.

e) If a matrix A has the QR factorization $A = QR$ then the equation $R\vec{x} = Q^T\vec{b}$ has a solution for any $\vec{b}$.